Stability, Specialization and Social Recognition*

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Abstract

Yang's theory of economic specialization under increasing returns to scale (Yang 2001) is a formal development of the fundamental Smith-Young theorem on the extent of the market and the social division of labor. In this theory specialization—and, thus, the social division of labor—is firmly embedded within a system of perfectly competitive markets. This leaves unresolved whether and how such development processes are possible in economies based on more primitive, non-market organizations.

In this paper we introduce a general relational model of economic interaction. Within this non-market environment we discuss the emergence of economic specialization and ultimately of economic trade and a social division of labor. We base our approach on three stages in organizational development: the presence of a stable relational structure; the presence of relational trust and subjective specialization; and, finally, the emergence of objective specialization through the social recognition of subjectively defined economic roles.

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1 Of specialization, institutions and social organization

Xiaokai Yang visited the Center for Economic Research at Tilburg University during the Spring of 1999. Immediately he engaged two of the three authors in extensive discussions on his research program. We easily identified similar research interests and this led to some fruitful exchanges and discussions.

During Professor Yang's visit to Tilburg we in particular discussed working paper versions of papers that were published subsequently as Diamantaras, Gilles, and Ruys (2003) and Sun, Yang, and Zhou (2004). These papers address some of the central problems and theoretical questions that lie at the core of our respective research programs. It is therefore fitting that in this paper we return to these central questions and sketch a joint research program that addresses both sets of questions.

The research program of Professor Yang was seminally developed in Yang (1988) and subsequently brought to fruition in numerous research papers.[1] The core of this research program is the application of inframarginal analysis to the decision model underlying a consumer-producer within a system of perfectly competitive market. In turn, this approach is used to model the Smith-Young approach to the relationship of specialization, the social division of labor, and increasing returns to scale. (Smith 1776, Young 1928, Stigler 1951)

Smith (1776) argued in his seminal work *Wealth of Nations* that the social division of labor is limited by the extent of the market so that the benefits of specialization to an individual are determined largely by the existing social division of labor in the economy. (This is also known as the *Smithian Theorem*.) Young (1928) extended this into a synergetic argument that the extent of the market also depends upon the level of social division of labor. Thus, the presence of increasing returns to scale leads to specialization and further social division of labor. In turn, a high level of social division of labor leads to increasing economies of specialization that form further incentives to specialize and deepen the social division of labor.

In the present paper we intend to sketch an argument that extends the Smithian theorem beyond the setting of a competitive market economy based on a system of perfectly competitive markets. Our argument is that the Smith-Young mechanism also applies to social organizations and institutional settings other than that of a system of perfectly competitive markets.

Indeed, we argue that the process of specialization occurs at different levels of embeddedness of the individual consumer-producer and that only at its most advanced state—namely that of objective specialization—this process results into a social division of labor. Thus, a social division of labor can indeed exist and generate economic development and growth in the context of more primitive economic institutions

[1] We refer to Yang (2001), Yang (2003) and Cheng and Yang (2004) for a comprehensive review of the work that has been accomplished in this research program.

and systems of imperfectly competitive markets. This development mechanism is *not* based on the endogenous selection of a specialization by an individual based on the prevailing market prices; instead, each individual selects from a given set of social economic roles, each corresponding to some specialism. Each of these social economic roles is collectively recognized as such and, regarding each of these social roles, there is a common knowledge.

Yang and Borland (1991) already showed that the Smith-Young mechanism functions as a determining factor in economic growth. Indeed, the mechanism of ever-deepening economic specialization and the accompanying development of the social division of labor leads to significant growth. In economic history and the new institutional economics this has been accepted as the main engine behind the rise of the western economies. (North and Thomas 1973, North 1990, Greif 1994, North 2005)

Recently, Ogilvie (2004), Acemoglu, Johnson, and Robinson (2005) and Gref (2006) have extended this argument and pointed to economic organizations other than the perfectly competitive market in which the Smith-Young mechanism causes economic development and growth. Acemoglu, Johnson, and Robinson (2005) mainly point to the development of property rights and the underlying political institutions as causes of economic growth. Empirical evidence of past performance of western economies back up these arguments.

In the current paper we present a model of a rather primitive economy in which economic agents directly interact with each other without reference to a central organization such as a system of competitive markets. Instead individual economic agents engage in binary, value-generating relationships—or "matchings". Matchings have to be understood as binary productive engagements, which are not necessarily trade relationships. It is assumed in this very primitive economy that every individual activates exactly one value-generating matching.

Our theory is developed along two different lines of thought. The first line is that of a formal theory in which we develop precise mathematical definitions and show two main theorems. The first theorem gives conditions under which equilibrium in a specific matching economy can be sustained; the second theorem gives a generic existence result that supports the emergence of a social division of labor.

The second line of thought develops an application of our theory to a specific case to illustrate the notions of subjective and objective specialization. Our main argument is that there are two different types of stability possible within a matching economy.

Subjective stability: Individuals engage in binary value-generating relationships—or matchings—and stability is attained if individuals are not willing to become autarkic or switch partners for higher benefits. The presence of stability is thus "subjective" in the sense that it is completely based on the properties of the productive abilities and utility functions of the individuals in the economy.

If a state of subjective stability is attained in the economy, the individuals might develop mutually beneficial trade within the relationship that they are engaged in. Moreover, individuals might specialize their productive activities within the (subjective) setting of the matching that they are engaged in. This is called *subjective* specialization since it is founded on the specific properties of the matching in which they generate their utilities.

We emphasize that subjective specialization does *not* induce a social division of labor since individuals are not engaged at a higher social plane; their economic interaction is explicitly limited to be within their matchings only. In that regard the organization of the economy remains scattered and there are no widespread gains from trade.

Generic stability: Only if generic stability is feasible, economic agents can truly specialize in an objective fashion and there emerges a social division of labor. A matching economy attains generic stability if for *every profile* of utility functions and production sets, there exists a stable matching pattern. Our main theorem states that such generic stability is attained if there is a social organization of the economy based on at least two socially recognized roles. Hence, there exist two socio-economic roles and value-generating relationships solely exist between individuals with different social roles. Only after the social roles of hunter and gatherer are established, a true endogenous social division of labor can emerge in which individuals specialize either as a hunter or a gatherer.

Our main existence theorem on generic stability thus identifies that a binary social division of labor is a pre-requisite for stability. This amends the Smithian theorem in the sense that there has to exist a finite set of socio-economic roles into which individuals can specialize, to establish stability in the social organization of the economy. The emergence of a set of socially recognized roles is, thus, a necessary condition for stability in the economy.

Hence, economic prosperity is determined largely by the set of available social roles in the economy. The Smith-Young mechanism of economic development can now be linked to the development of this set of socially recognized roles; innovation in social organization—in the sense that new social roles are developed—now determines the extent of the market and, thus, economic growth.

Although our model of a matching economy describes a very primitive society, we believe that it makes possible some deep conclusions. We believe that our approach also resolves the indeterminacy problem identified by Gilles and Diamantaras (2005). They argued that the theory of the Smith-Young development mechanism is founded on a circular argument: prices of traded goods determine individuals' specialization and, thus, prices determine the social division of labor. This, in turn, determines which goods are produced and traded, thus determining the extent of the market. This brings up the question who or what ultimately determines which goods are traded and how economic development is accomplished.

In our current model we put this determinacy problem at the center of our analysis. Indeed, our main result states that generic stability requires the existence of a certain set of established social roles from which individuals can choose when they specialize. Each social role stands for a certain social-economic specialization and in equilibrium the number of agents of each role is balanced. Only then an effective social division of labor emerges and the society can engage into an effective process of economic development and growth. Ultimately this development is founded on the enhancement and extension of the commonly known set of economic roles.

Ultimately we conclude that economic development and growth is caused by organizational and institutional change (Acemoglu, Johnson, and Robinson 2005), rather than technical change only (Romer 1986, Romer 1990). We believe that technical change is a consequence and expression of the effectiveness of the social organization of the economy.

In Section 2 we develop our model of a matching economy based on binary value-generating activities among economic agents. In Section 3 we define stability as our main equilibrium notion and develop the application to a primitive hunter-gatherer economy. Section 4 discusses the existence of stable matching patterns and the emergence of subjective specialization. In Section 5 we introduce generic stability and the possibility of objective specialization. This in turn implies the emergence of a social division of labor in such a matching economy. We summarize our main line of thought in Section 6.

2 Matching economies

Let $N = \{1, ..., n\}$ be a finite set of *individuals*. At this stage we do not make any assumptions about these individuals regarding their individual abilities. Hence, in this general model we do not explicitly assume that these individuals are consumer-producers or that they are even able to specialize in any form.

Instead we endow these individuals with the abilities to engage into relational economic activities that generate economic values or wealth.[2] Therefore, these individuals are assumed to have *relational* abilities. (These relational abilities have to be understood as special forms of more generalized social-economic abilities.) These relational abilities in turn might be based on individualistic abilities; this approach

[2]The most primitive form of a matching is that of cooperation in some production activities. More advanced forms include the simple exchange or *trade* of two commodities. The gains from trade then form the values that are generated between the two traders.

is explored in some examples throughout this paper. Note that we do not assume or impose that these relational activities take place in the context of a market. Instead we assume that these relational abilities describe the economy itself.

Formally, we let $\Gamma \subset \{ij \mid i, j \in N\}$ be a set of potential relational activities between the individuals in N. Here, for two distinct individuals $i \in N$ and $j \in N$ with $i \neq j$ we define by $ij \in \Gamma$ that these individuals i and j are able to engage in a "value-generating relational activity". We indicate this potential relational engagement $ij \in \Gamma$ as a *potential matching* of i and j. This is formalized as follows.

Definition 2.1 *A* ***potential matching structure*** *on the set of individuals* N *is given as* $\Gamma \subset \{ij \mid i, j \in N\}$ *such that*

(i) *for every individual* $i \in N$: $ii \in \Gamma$ *and*

(ii) *for every individual* $i \in N$ *there exists some* $j \in N$ *with* $j \neq i$ *and* $ij \in \Gamma$.

Every relationship ij *in the structure* Γ *on* N *is denoted as a* ***potential matching****.*

We emphasize that any potential matching is symmetric in the sense that a matching between individuals i and j is exactly the same matching as the one between individuals j and i. On the other hand, individuals i and j need not have the same utility from this potential matching, as it will become evident later.

It is also possible that an individual $i \in N$ does not engage in an economic activity with any of the other economic individuals. In this regard i attains a *relationally autarkic position*.[3] Mathematically this is represented by the pairing of i with himself, i.e., by the matching ii. The definition of Γ assumes that each player $i \in N$ has the possibility to exclude himself from the relational activities in this economy and assume a relationally autarkic position, indicated by $ii \in \Gamma$. We define

$$\Gamma_0 = \{ii \mid i \in N\} \subset \Gamma$$

as the collection of relationally autarkic positions.

Another interpretation is that the potential matching structure Γ represents the social capital that is present within the population N. It describes what is the potential set of matching partners for each individual, i.e., the complete description of her potential social interactions. Some of these potential interactions may generate positive utilities and others negative. Most importantly, it is assumed that no two individuals i and j with $ij \notin \Gamma$ can even engage in an economic value-generating relation. This indeed corresponds to the notion of social capital as used in the social sciences. (Por 1998, Putnam 2000, Dasgupta 2005)

[3]Throughout the paper we distinguish two types of autarky: relational autarky and trade autarky. *Relational autarky* refers to the state of isolation of a player within the structure of all potential relations Γ, while *trade autarky* refers to a state of nonparticipation in any of the trade processes in the economy. Obviously, relational autarky implies trade autarky.

The relative position of an individual in Γ defines his matching possibility set as it will become clear in the analysis. Formally, for every potential matching structure Γ and every individual $i \in N$, we introduce i's *neighborhood* in Γ as the set of individuals who can be partners of player i in potential matchings, i.e.,

$$N_i(\Gamma) = \{j \in N \mid ij \in \Gamma \text{ with } i \neq j\}.$$

The set of potential matchings that individual i can engage in, can now be formulated as

$$L_i(\Gamma) = \{ij \in \Gamma \mid j \in N_i(\Gamma)\}.$$

Let $m \in \mathbb{N}$. A *path* between individuals i and j in the potential matching structure Γ is a set of distinct individuals $P(ij) = \{i_1, i_2, \ldots, i_m\} \subset N$ such that $i_1 = i$, $i_m = j$ and $i_k i_{k+1} \in \Gamma$ for all $k \in \{1, \ldots, m-1\}$. The *length* of the path $P(ij)$ is said to be the number of links $m-1$ that make up this path.

A *cycle* in the structure Γ is a set of distinct players $C = \{i_1, i_2, \ldots, i_m\}$ with $m \geqslant 4$ such that $i_1 = i_m$ and $i_k i_{k+1} \in \Gamma$ for all $k \in \{1, \ldots, m-1\}$. Now the *length* of the cycle C is given as $m-1$. Thus, a cycle is a path from an individual to herself, which consists of at least three distinct players. We emphasize that each cycle has length of at least three, i.e., a cycle consists of at least three distinct relations.

Definition 2.2 *We say that a sub-structure $\Omega \subset \Gamma$ of the potential matching structure Γ on N is **odd acyclic** if Ω does not contain any cycle C of length $\ell \geqslant 3$ such that ℓ is an odd integer.*

Odd acyclicity turns out to be a crucial property in the further development of our theory.

To complete our model we assume that every individual $i \in N$ is endowed with complete and transitive preferences over the potential matchings $L_i(\Gamma) \subset \Gamma$ in which she can engage in. Thus, by finiteness of Γ, these preferences can be represented by a *hedonic utility function* given by $u_i \colon L_i(\Gamma) \to \mathbb{R}$. Let $u = (u_1, \ldots, u_n)$ denote a *hedonic utility profile* and $\mathcal{U}$ be the set of all hedonic profiles representing complete and transitive preferences.

Definition 2.3 *A **matching economy** is defined to be a triple $\mathbb{E} = (N, \Gamma, u)$ in which N is a finite set of individuals, Γ is a potential matching structure on N, and $u \in \mathcal{U}$ is a hedonic utility profile on Γ.*

A matching economy essentially is based on potential binary activities that generate economic values. For example, a trade economy can be represented as a matching economy between buyers and sellers who can trade physical goods to generate gains from trade. We emphasize here that a trade economy with two commodities—one *desirable* and *money*—imposes that the potential matching structure Γ is bipartite and that there are in fact two social types of individuals, namely buyers of the desirable and sellers of the desirable. This in turn implies that Γ is odd-acyclic. This imposes very strong properties on the matching economy as we explore in subsequent sections of this paper.

Similarly, as an application to the health sector of an economy, by performing surgery on a patient a physician generates a significant increase of the life expectancy of that patient, thus generating a social value. There can also be examples when the roles are not specified such as pairing students to work on a class project.

3 Relational stability and equilibrium

Within the context of a matching economy we investigate the proper definition of stability. A stable interaction pattern is crucial to further develop our theory. Stability is a necessary condition for the further development of an economy, in particular for the emergence of specialization and a social division of labor.

Our main hypothesis in our definition of stability is that in a matching economy $\mathbb{E} = (N, \Gamma, u)$ each individual activates *exactly one* of her potential matchings. This fundamental hypothesis is founded on the fact that we model a very primitive economy without the presence of advanced economic or social institutions. In such a primitive economy it is natural to assume that individuals only interact with a single other individual at a time and that more complex interactions require more advanced social institutions than assumed within our context.

Within the confines of the constraints imposed on the potential economic activities of the individuals in the economy $\mathbb{E}$ we now define a matching pattern as a set of activated potential matchings.

Definition 3.1 *A* ***matching pattern*** *is a subset of the potential matching structure $\pi \subset \Gamma$ such that every individual is either paired with exactly one other individual or remains relationally autarkic, i.e., $\pi \subset \Gamma$ is such that $|L_i(\pi)| = |N_i(\pi)| = 1$, for all $i \in N$.*
We denote by $\Pi(\Gamma) = \Pi$ the class of all potential matching patterns within Γ.

In a matching pattern one and only one matching is selected and executed by each individual. For ease of notation we denote the utility an individual i has when participating in a matching pattern π in which $L_i(\pi) = \{ii_\pi\}$ as $u_i(\pi)$, i.e., $u_i(\pi) \equiv u_i(ii_\pi)$, for all $i \in N$.[4]

With the tools developed so far we are able to introduce two relational stability concepts. Again we let the matching economy $\mathbb{E} = (N, \Gamma, u)$ be given throughout. For

[4]We emphasize that the hedonic utility profile considered here allows an individual to consider only one matching at a time, since we do not allow an individual to engage in multiple matchings at the same time.

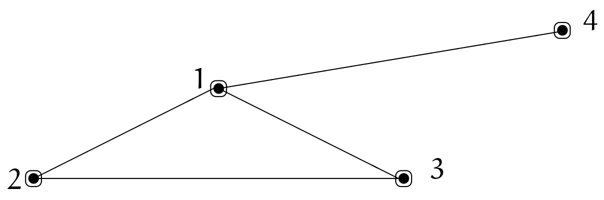

Figure 1: The potential matching structure G in Example 3.3.

matching pattern $\pi \in \Pi$, a potential matching $ij \in \Gamma \setminus \pi$ is a *blocking matching* if $u_i(ij) > u_i(\pi)$ as well as $u_j(ij) > u_j(\pi)$.

Having defined a blocking matching as a *strict* binary Pareto improvement, we follow the concepts used in the literature on matching (Roth and Sotomayor 1990). We point out that our notion of stability is closely related to that of stability in network formation (Jackson and Wolinsky 1996). With this concept we can define our stability property of a matching pattern.

Definition 3.2 *A matching pattern $\pi \in \Pi$ is **stable** in the economy $\mathbb{E} = (N, \Gamma, u)$ if all matchings in π satisfy the **individual rationality** (IR) and **no blocking** (NB) conditions:*

IR $u_i(\pi) \geqslant u_i(ii)$ *for all* $i \in N$, *and*

NB *there is no blocking matching with regard to π, i.e., for all $i, j \in N$, $i \neq j$, with $ij \in \Gamma \setminus \pi$:*

$$u_i(ij) > u_i(\pi) \text{ implies that } u_j(ij) \leqslant u_j(\pi). \tag{1}$$

Stable matching patterns in $\mathbb{E}$ are denoted by $\pi \in \Pi^\star(N, \Gamma, u)$.

Condition (IR) is an individual rationality requirement, that states that an individual cannot be matched with another individual without her consent, i.e., if an individual is better-off under relational autarky, she will pursue that.

In (NB) stands for a non-blocking condition requiring that a blocking matching does not exist with respect to matching pattern $\pi \in \Pi$. Under (NB) if an individual prefers to be matched with an alternative individual than the one with whom he is matched under matching pattern π, then that alternative individual does not agree to engage with him. This condition is closely related to the condition of link addition proofness in network formation. Link addition proofness is at the foundation of the notion of pairwise stability in network formation, seminally introduced by Jackson and Wolinsky (1996).

To illustrate our definition of stability, we discuss an abstract example.

Example 3.3 Consider an economy $\mathbb{E}_1 = (N, \Gamma, u)$ with $N = \{1, 2, 3, 4\}$, potential matching structure $\Gamma = \{12, 23, 13, 14, 11, 22, 33, 44\}$, and the utility profile u given in the table below.[5]

$j =$	1	2	3	4
$u_1(1j)$	0	1	2	3
$u_2(2j)$	1	0	2	–
$u_3(3j)$	2	-1	0	–
$u_4(4j)$	0	–	–	1

Given Γ we now derive the collection of all possible matching patterns, which is given by

$$\Pi = \{\{11, 22, 33, 44\}; \{11, 23, 44\}; \{12, 33, 44\}; \{13, 22, 44\}; \{23, 14\}\}.$$

We now identify the stable matching patterns in this example. Let us start the discussion with individual 1. She prefers to be matched to individual 4 since her utility in this matching is the highest. However, individual 4 prefers to be by herself rather than to be matched with 1 ($u_4(14) < u_4(44)$). Hence a matching between individuals 1 and 4 violates the individual rationality condition for individual 4.
Excluding link 14, individual 1 prefers to be matched with individual 3. Since individual 1 is also individual 3's most preferred partner, a matching between them cannot be blocked by individual 2. Finally, individuals 2 and 4 do not have a potential matching, hence in the matching pattern they should be in a state of relational autarky. Therefore, the unique stable matching pattern is given by $\pi^* = \{13, 22, 44\}$. □

Our main application of the general relational framework developed is that of a relational economy of consumer-producers. We follow the new classical framework developed in Yang (2001) and Yang (2003). The new classical approach is firmly founded on the premise that consumer-producers specialize within a social context of a structure of (market) interactions and, thus, attain higher welfare levels.

Here we start at an even more primitive level of reasoning. Before there is actual specialization, there are consumer-producers with simple *skills* on which these specializations can be based. We recognize that skills, unlike commodities, are intrinsic to a consumer-producer and cannot be exchanged. They can, however, be shared. Sharing one's skills with another individual is a process that does not make the giver any poorer in the skill.[6] As established by Yang and Borland (1991) and Yang (2003), learning-by-doing is an important mechanism in the process of growth. However, in Yang's framework this process is individual-specific, i.e., economic individuals are not allowed to learn from each other. In our framework, we go beyond this restriction

[5] In this table a dash in a cell indicates that no potential matching between individuals i and j exists.

[6] A commodity, in comparison, if shared makes the giver poorer in the possession of that commodity. This is to say that while commodities are pure private goods, skills are non-rival in nature.

by allowing limited learning between individuals. When two individuals engage in a relational activity, they do not actually exchange consumption goods, as in the case of Yang; instead their learning externalities increase their productivity through the (limited) sharing of the skills accumulated by their partners.

These ideas are illustrated in Example 3.4 below. There is a finite set of consumer-producers. Each individual is endowed with one unit of productive time. There are two types of skills, hunting (H) and gathering (G), complementing the production of two types of consumption goods, meat and vegetables such as roots and corn. When individuals are engaged in a matching they acquire also some of the skills acquired by their partner. Thus, there are relational externalities in the acquisition of skills.

Individuals principally engage in the individual accumulation of hunting and gathering skills. We also implement that they can decide to match with another individual and enjoy the relational externalities in the acquisition of skills with this other individual; skills are actually shared. This sharing is based on some learning process between the matched individuals. Within such a "sharing" matching, each individual produces meat and vegetables by hunting and gathering, respectively. Before making a decision to match, each individual can calculate the potential production and level of utility attainable in each potential matching.

At this point in the development of a society, it is *not* assumed that matched individuals actually engage in the exchange of the produced goods if this is beneficial for both parties. Instead they remain trade autarkic[7] and only share their skills in the way described above.

Example 3.4 (A relational economy with consumer-producers)
Let $N = \{1, 2, 3\}$ be the set of three individuals. Each individual is endowed with one unit of time which she can use to acquire gathering skills G_i and hunting skills H_i. Skill acquisition is linear in time, i.e., $G_i = l_i$ and $H_i = 1 - l_i$ where $l_i \in [0, 1]$ is the labor time used by individual i in acquiring gathering skills G_i. Each individual i is therefore endowed with a technology to produce two types of consumption goods: vegetables (v) and meat (m) by using gathering skills G_i and hunting skills H_i, respectively.
Furthermore, the interaction between these individuals is introduced as a complementarity in skill acquisition; individuals can acquire some of the skills of their matching partner. This is described by two *learning parameters* $\alpha^i_{ij}, \beta^i_{ij} \in [0, 1]$, which are individual and pair specific. The parameters α (respectively β) describe the transfer of gathering (respectively hunting) skills from an individual's partner to that indivi-

[7] As introduced before, we use the term "trade autarkic" to express that an individual is self-sufficient without engaging in trade to obtain certain commodities.

dual. The corresponding production functions are now introduced as

$$g_i(ij) = (G_i(1+\alpha^i_{ij}G_j))^2 \quad \text{and}$$
$$h_i(ij) = (H_i(1+\beta^i_{ij}H_j))^2 \quad \text{for all } i,j = 1,2,3.$$

In this example we assume that the learning parameters are given in the following table:

ij	α^i_{ij}	α^j_{ij}	β^i_{ij}	β^j_{ij}
11	0	0	—	—
12	0.3	0.6	0.3	0.6
13	0.5	0.3	0.8	0.3
22	0	0	—	—
23	0.3	0.8	0.3	0.5
33	0	0	—	—

Individuals are endowed with homothetic preferences over the consumption of meat and vegetables given by

$$u_i(\nu_i\, m_i) = \sqrt{\nu_i\, m_i} \tag{2}$$

where ν_i denotes the consumption of vegetables by individual i and m_i denotes the consumption of meat by individual i.

The optimal acquisition of skills

The optimal investment in hunting and gathering skill of each individual depends on the specialization decisions made by other individuals. First, we consider the case in which individuals maximize their utility in the relationally autarkic case.[8] The relationally autarkic utility maximization problem is given by

$$\max_{0\leqslant l_i\leqslant 1} u_i(\nu_i(l_i)\, m_i(l_i)) = l_i(1-l_i) \quad \text{for all } i = 1,2,3.$$

The solution yields $l_i = \frac{1}{2}$ for all individuals $i = 1,2,3$. Hence, they invest equally in acquiring gathering and hunting skills, i.e., $G_i = H_i = \frac{1}{2}$.

Second, given the externality parameters α and β, we can calculate the optimal investment of an individual in acquiring hunting and gathering skills *given the skill levels of her partner*. To take a generic case, let the partner j of individual i have acquired skill levels H_j and G_j respectively. Then the utility maximization problem of individual i is given by

$$\max_{0\leqslant l_i\leqslant 1} u_i(\nu_i(l_i)\, m_i(l_i)) = \left[l_i(1+\alpha^i_{ij}G_j)\right]\cdot\left[(1-l_i)(1+\beta^i_{ij}H_j)\right] \tag{3}$$

[8] This captures the extremely pessimistic case in which individuals believe that they cannot match to any other individual. This can also be considered to be the outcome of the maximization problem of extremely risk-averse individuals, or individuals who have very low degree of trust in the abilities of the other individuals.

Irrespective of the parameter values α^i_{ij} and β^i_{ij} and of the levels H_j and G_j, this reduces to the same optimization problem as under relational autarky. Thus, individuals remain trade autarkic irrespective of the complementarities in the relationships with their partners. So, again the optimal investment in acquisition of skills is given by $l_i = \frac{1}{2}$ implying that $H_i = G_i = \frac{1}{2}$.

The resulting matching economy

Given the optimal acquisition of skills, we first compute the optimal production outputs for vegetables and meat for all potential relationships. Subsequently, we determine the resulting potential utility values.

In fact, given $H_i = G_i = \frac{1}{2}$ for all individuals $i = 1, 2, 3$, the potential production levels of meat and vegetables by each individual in each possible matching are now given by

ij	$g_i(ij)$	$h_i(ij)$	$g_j(ij)$	$h_j(ij)$
11	0.25	0.25	—	—
12	0.3306	0.3306	0.4225	0.4225
13	0.3906	0.49	0.3306	0.3306
22	0.25	0.25	—	—
23	0.3306	0.3306	0.49	0.3906
33	0.25	0.25	—	—

We emphasize again that, since all individuals remain trade autarkic, no trade will ensue. Moreover, note that there is no mutually beneficial trade between any two individuals because in any pair one of the individuals has bigger quantities of both goods. In fact, we assume that all individuals believe that they will not engage in trade after creating a relationship with another individual.[9] Hence, we can calculate the utilities profile in a straightforward way, e.g., $u_1(13) = \sqrt{g_1(13) \cdot h_1(13)} = \sqrt{0.3906 \times 0.49} = 0.4375$. Similarly, the remainder of all utility profiles is computed and presented in the table below.

j	1	2	3
$u_1(1j)$	0.25	0.3306	0.4375
$u_2(2j)$	0.4225	0.25	0.3306
$u_3(3j)$	0.3306	0.4375	0.25

.

The absence of stability

We claim that in the resulting matching economy, there does *not* exist a stable matching pattern. Hence, in this economy based on the acquisition of complementing skills, there does not exist an equilibrium.

[9] As argued in the introduction, trade can only emerge within stable relations. Thus, only within a stable matching pattern such trade can evolve. We also refer to Examples 4.7 and 4.8 for further details.

As the utility profile shows, individual 1 prefers to be matched with individual 3 rather than with individual 2. Individual 2 prefers to be matched with individual 1 rather than with individual 3. Individual 3 prefers to be matched with individual 2 rather than with individual 1. Finally, all individuals prefer to be matched with a partner rather than to stay relationally autarkic. Hence, we conclude that there is no stable matching pattern. □

4 Existence of stability and subjective specialization

In the previous discussion, we have shown that in a primitive economy with limited specialization, there might be no equilibrium emerging in the form of a stable matching pattern. Here we investigate the sufficient conditions for the existence of stable matching patterns. We also discuss the implications of our findings with regard to specialization in a relational economy.

Our analysis requires the introduction of several auxiliary notions. We define for any sub-collection of matching patterns $\Theta \subset \Pi$ its *cover* by

$$\overline{\Theta} = \left(\bigcup_{\pi \in \Theta} \pi \right) \setminus \Gamma_0. \tag{4}$$

where $\Gamma_0 = \{ii \mid i \in N\} \subset \Gamma$ denotes the set of relationally autarkic relationships.

Below we define a specific subclass of matching patterns. A similar class of matchings has been defined by Sotomayor (1996) in her proof of existence of stable matching patterns in a bipartite matching economy. (Sotomayor refers to these patterns as "simple"; we do not adopt this terminology.)

Definition 4.1 *A matching pattern $\pi \in \Pi$ is **weakly stable** in $\mathbb{E} = (N, \Gamma, u)$ if for all individuals the Individually Rationality (IR) condition holds and whenever a blocking matching $ij \in \Gamma \setminus \pi$ exists, at least one of the partners in ij is relationally autarkic under π, i.e.,*

$$u_i(ij) > u_i(\pi) \textit{ and } u_j(ij) > u_j(\pi) \textit{ imply that } \{ii, jj\} \cap \pi \neq \varnothing. \tag{5}$$

We denote this as $\pi \in \Pi_w(N, \Gamma, u) = \Pi_w \subset \Pi$.

In a weakly stable matching pattern at least one of the partners in a blocking matching is autarkic, hence if we are to delete all the relationally autarkic individuals from such a pattern the remaining matchings will be stable. Further, note that the set of weakly stable matching patterns Π_w is non-empty as it contains at least the autarkic matching pattern $\Gamma_0 = \{ii \mid i \in N\} \subset \Gamma$. We use these properties of Π_w to show the existence of stable matching patterns.

We first establish the following trivial insight, which follows immediately from Definitions 3.2 and 4.1.

Lemma 4.2 *Every stable matching pattern is also weakly stable.*

We recall the definition (4) of the cover of $\Pi_w(N, \Gamma, u)$ as

$$\overline{\Pi}_w = \left[\left(\bigcup_{\pi \in \Pi_w} \pi \right) \setminus \Gamma_0 \right] \subset \Gamma. \tag{6}$$

Similar to the odd acyclicity property of the set of potential matchings, we define odd acyclicity property of a matching economy $\mathbb{E}$.

Definition 4.3 *A matching economy $\mathbb{E} = (N, \Gamma, u)$ is* ***odd acyclic*** *if for the class of weakly stable matching patterns $\Pi_w(N, \Gamma, u)$ it holds that its cover $\overline{\Pi}_w \subset \Gamma$ defined in equation (6) is odd acyclic.*

We first show that it is possible that the class of all permissible matching patterns Π is not odd acyclic—and, thus, $\overline{\Pi} \equiv \cup_{\pi \in \Pi} (\pi \setminus \Gamma_0) = \Gamma \setminus \Gamma_0$ contains a cycle of odd length—while the economy $\mathbb{E}$ itself is odd acyclic.

Example 4.4 Consider the matching economy $\mathbb{E}_1$ given in Example 3.3. Now in $\mathbb{E}_1$ the cover $\overline{\Pi}$ of the collection of possible matching patterns consists of a cycle with odd number of links, between individuals 1, 2, and 3. Indeed, $\{12, 23, 31\} \subset \overline{\Pi} = \{12, 13, 14, 23\}$.
On the other hand, given the utility profile u, the set of weakly stable matching patterns Π_w is given by

$$\Pi_w = \{\{11, 22, 33, 44\}; \{12, 33, 44\}; \{13, 22, 44\}\}.$$

We now see that $\mathbb{E}_1$ is odd acyclic. Indeed, $\overline{\Pi}_w = \{12, 13\}$ and therefore does not contain a cycle. □

Our main existence theorem states that stable matching patterns exist if the collection of weakly stable matching patterns satisfy the odd acyclicity condition. We refer to Chung (2000, Theorem 1) for a similar result for the case of a pure matching problem.[10]

Theorem 4.5 *If a matching economy $\mathbb{E} = (N, \Gamma, u)$ is odd acyclic, then it holds that $\Pi^\star(N, \Gamma, u) \neq \varnothing$.*

Proof. First, we consider the case that the cover of the collection of weakly stable matching patterns $\overline{\Pi}_w$ does not contain *any* cycle. Subsequently, we investigate the case that $\overline{\Pi}_w$ only contains cycles that have an even number of links.

A: $\overline{\Pi}_w$ IS ACYCLIC.

[10] In his stability result Chung (2000) imposes the odd-acyclicity condition on the preference profile of the agents.

Assume that $\overline{\Pi}_w$ does not contain any cycle, and suppose that no stable matching pattern exists. Then for any weakly stable matching pattern $\pi \in \Pi_w$ there is a blocking matching. By definition of a weakly stable matching pattern in such a blocking matching at least one of the individuals is relationally autarkic under π. Hence, without loss of generality, we can take a weakly stable pattern $\pi \in \Pi_w$ for which ij is a blocking matching, $ii, jk \in \pi$, and there is a match of i with j leaving k alone and keeping all other matchings the same. Matching pattern π', obtained in this way, must be weakly stable, i.e., $\pi' \in \Pi_w$ since there can be only one new blocking matching and it contains individual k, who is relationally autarkic under π'.

Since π' is not stable, individual k can form a blocking matching with another individual, say l, such that $lk \notin \pi'$. By forming the pair kl, a new matching pattern is formed $\pi'' = \pi' \cup \{kl\} \in \Pi_w$. Note that $l \neq i$ since Π_w does not contain a cycle. Now the matching pattern π'' can in turn be blocked by a matching ps where $ps \notin \pi''$. Thus, a new matching pattern $\pi''' = \pi'' \cup \{ps\} \in \Pi_w$ is generated where $p \neq j$, since $\overline{\Pi}_w$ does not contain a cycle.

Iterating a sequence of matching patterns $\pi^{(k)}$ with $k \in \mathbb{N}$ according to the construction outlined above, we reach a contradiction to the acyclicity due to the finiteness of the number of individuals.

B: $\overline{\Pi}_w$ IS CYCLIC AS WELL AS ODD ACYCLIC.

Next we assume that $\overline{\Pi}_w$ is odd acyclic. Let $\overline{\Pi}_w$ consist of a single cycle, i.e., $\overline{\Pi}_w = \{i_1i_2, i_2i_3, ..., i_{k-1}i_k\}$ such that $i_k = i_1$, $k \geqslant 3$, and $k-1$ is an even integer.

We consider two cases, distinguished by the preference profile of the individuals. In the first case the proof of the existence of a stable matching pattern is reduced to the analysis of an acyclic cover of the collection of weakly stable matching patterns. In the second case we propose an algorithm and prove that it leads to identifying a stable matching pattern.

Case I: $\overline{\Pi}_w = \{i_1i_2, i_2i_3, ..., i_{k-1}i_k\}$, $i_k = i_1$, $k \geqslant 3$, and $k-1$ is an even integer and let there be at least one pair $ij \in \overline{\Pi}_w$, such that individual i is in the set of most preferred partners of individual j and individual j is in the set of most preferred partners of individual i, i.e., $j \in B_i(\Gamma) \equiv \{k \in N_i(\Gamma) \cup \{i\} \mid u_i(ik) \geqslant u_i(ih) \text{ for all } h \in N_i(\Gamma) \cup \{i\}\}$ and $i \in B_j(\Gamma)$. Note that individual i is not necessarily different from individual j. However, if $i = j$, then the set of individual i's most preferred partner must contain also his two neighbors along a cycle.

Then it follows that ij is an element of any stable matching pattern, otherwise it will form a blocking matching. Next consider the set of weakly stable matching patterns which does not contain the pair ij. Thus truncated, the cover of the class of weakly stable matching patterns, $\overline{\Pi}_w \setminus ij$, is acyclic and the existence of a stable matching pattern, $\pi^\star$, follows from discussion of the first part of the proof.

Case II: Assume that $\overline{\Pi}_w = \{i_1i_2, i_2i_3, \ldots, i_{k-1}i_k\}$, $i_k = i_1$, such that there is no mat-

ching ij for which $j \in B_i(\Gamma)$ and $i \in B_j(\Gamma)$. Note that this precludes any of the individuals from having relational autarky as the most preferred state.
Without loss of generality consider a preference profile $u = (u_{i_1}, \dots, u_{i_{k-1}})$ such that $u_{i_s}(i_s i_{s+1}) > u_{i_s}(i_{s-1} i_s)$, for all $s = 1, \dots, k-1$ where $i_0 = i_{k-1}$. Consider the following algorithm for selecting a matching pattern:

> Take any individual $i_s \in \{1, \dots, k-1\}$ and match her with her most preferred partner, $B_{i_s} = i_{s+1}$, hence $i_s i_{s+1} \in \pi$;
> Then consider the most preferred partner of individual i_{s+1} and match him with his most preferred partner, i.e., $B_{i_{s+1}}$ is i_{s+2}, and $B_{i_{s+2}} = i_{s+3}$, therefore $i_{s+2} i_{s+3} \in \pi$;
> Continue until all individuals are matched in π. Note that all individuals in π are in a matching with another individual, thus, $\pi \in \Pi_w$ if and only if π is stable.

Now, suppose that π is not a stable matching pattern. Then there exists a blocking matching $i_s i_{s+1}$ for $s = 1, \dots, k-1$ such that $u_{i_s}(i_s i_{s+1}) > u_{i_s}(\pi)$ and $u_{i_{s+1}}(i_s i_{s+1}) > u_{i_{s+1}}(\pi)$, which contradicts the construction of π in which one of every two consecutive individuals is matched with her most preferred partner in π. Thus, $\pi \in \Pi_w$ is a stable matching pattern. ■

In fact in the last case of the proof of Theorem 4.5 there are two distinct stable matching patterns. One is selected if the starting individual in the algorithm has an odd index on the cyclical path. The other stable matching pattern is selected if the starting individual in the algorithm has an even index on the cyclical path

The reverse of Theorem 4.5 is not necessarily true, i.e., if a stable matching pattern exists with respect to some $\Gamma \subset \Gamma_N$ then it might be that $\overline{\Pi}_w$ contains a cycle of odd length. This is illustrated in the following example.

Example 4.6 Consider again the matching economy $\mathbb{E}_1$ as discussed in Example 3.3 with the potential matching structure depicted in Figure 1. Now we modify the utility profiles over potential matchings as follows:

j	1	2	3	4
$u_1(1j)$	0	1	2	3
$u_2(2j)$	1	0	2	–
$u_3(3j)$	2	1	0	–
$u_4(4j)$	0	–	–	1

In this modified matching economy $\mathbb{E}_2$ there exists a unique stable matching pattern $\pi^\star = \{13, 22, 44\}$.[11] However, the cover of the set of simple matching patterns, $\overline{\Pi}_w$,

[11] We refer the reader to the discussion of Example 3.3 to see why this is a stable matching pattern.

generates a cycle. Indeed,

$$\Pi_w = \{\{11, 22, 33, 44\};\ \{13, 22, 44\};\ \{11, 23, 44\};\ \{12, 33, 44\}\}; \tag{7}$$

and therefore $\overline{\Pi}_w = \{12, 13, 23\}$ is an odd cycle itself. □

4.1 The development of trade

In Example 3.4 we showed that there might not exist stable matching patterns in relational settings with complementarities in skill acquisition. In such a matching economy, all individuals could establish mutually beneficial relationships with another individual based on relational complementarities in the acquisition of skills. However, in that example, the lack of mutual consent of most preferred partners precludes them from establishing these relationships. The absence of a stable matching pattern implies that there is essentially a state of chaos in such a society.

The next example extends the discussion in Example 3.4 and shows that in many cases there might emerge stable matching patterns within such situations. It develops a case of an economy in which the learning parameters allow the formation of a stable matching pattern consisting of mutually beneficial relationships.

Example 4.7 (Existence of stable matching patterns)
Consider the matching economy that has been developed in Example 3.4. We modify this example to allow the existence of a stable matching pattern. For this we modify the learning parameters as given in the table below:

ij	α^i_{ij}	α^j_{ij}	β^i_{ij}	β^j_{ij}
11	0	0	—	—
12	0.3	0.6	0.3	0.6
13	0.5	0.8	0.8	0.5
22	0	0	—	—
23	0.3	0.3	0.3	0.3
33	0	0	—	—

As in Example 3.4 individuals remain trade autarkic under the given circumstances and have an optimal investment in the acquisition of skills given by $l_i = \frac{1}{2}$. Hence, all individuals $i \in \{1, 2, 3\}$ attain skill levels $G_i = H_i = \frac{1}{2}$. This results into the following production levels:

ij	$g_i(ij)$	$h_i(ij)$	$g_j(ij)$	$h_j(ij)$
11	0.25	0.25	—	—
12	0.3306	0.3306	0.4225	0.4225
13	0.3906	0.49	0.49	0.3906
22	0.25	0.25	—	—
23	0.3306	0.3306	0.3306	0.3306
33	0.25	0.25	—	—

These production levels now result into the following potential utility levels:

j	1	2	3
$u_1(1j)$	0.25	0.3306	0.4375
$u_2(2j)$	0.4225	0.25	0.3306
$u_3(3j)$	0.4375	0.3306	0.25

.

It is clear that for the given potential utility levels there exists a stable matching pattern. Indeed, the generated matching economy is odd acyclic and the pattern $\pi^* = \{13, 22\}$ is stable. This stable matching pattern results into utility levels given by $u_1^* = u_3^* = 0.4375$ and $u_2^* = 0.25$. □

Only after stable matchings have been formed, individuals can engage in mutually beneficial trade within such relationships. Without the support of a stable relationship, there would neither exist nor emerge any trust among the individuals and therefore there would be no institutional basis for trade.

However, within a stable matching, trade is founded on a moderate level of trust and both individuals can be assumed to engage in trade. This is the subject of the next extension of Example 4.7:

Example 4.8 (Justification of trade)
Consider the matching economy discussed in Example 4.7. This matching economy admits a stable matching pattern $\pi^* = \{13, 22\}$. The only relevant stable matching that emerges within this pattern is 13. Both individuals 1 and 3 can indeed engage in mutually beneficial trade within this relationship.
Note that within 13, $g_1 = 0.3906$, $h_1 = 0.49$, $g_3 = 0.49$, and $h_3 = 0.3906$. It is clear that the trade resulting within the relationship 13 ultimately leads to final consumption levels given by

$$v_1 = v_3 = \tfrac{1}{2}\left(\, g_1(13) + g_3(13)\,\right) = 0.4403, \text{ and}$$
$$m_1 = m_3 = \tfrac{1}{2}\left(\, h_1(13) + h_3(13)\,\right) = 0.4403.$$

This in turn leads to after-trade utility levels given by $\hat{u}_1 = \hat{u}_3 = 0.4403 > 0.4375 = u_1^* = u_3^*$. Hence, there are mutual gains from trade within the stable relationship between individuals 1 and 3. □

4.2 The emergence of subjective specialization

Example 4.8 indicates that within a stable matching, there naturally emerges a moderate level of trust and, consequently, the possibility of mutually beneficial trade. If such a stable matching is sustained, individuals will identify that *specialization* of their skills leads to further deepening of the gains from trade. Indeed, after both parties engage in trade, individual 1 will identify that increasing his skill level in hunting will increase his meat production further. Similarly, individual 3 will identify the complementary effect of increasing her gathering skills to increase her vegetable production.

This implies that further deepening of the stable trade relationship between individuals 1 and 3 results into mutual specialization. We emphasize that this specialization is induced at the most primitive level by the nature of the complementarities between these individuals. Indeed, that individual 1 specializes in hunting is a consequence of $\alpha^1_{13} < \beta^1_{13}$. Hence, there are social foundations to this specialization; specialization is still founded on the specific interaction within the relationship between 1 and 3. In this regard this type of specialization is completely *subjective*; this specialization only occurs within the context of the matching 13 and has no consequences beyond that relationship.

Another formulation of the foundation of such subjective specialization is to say that there are Ricardian comparative advantages for individual 1 to specialize in hunting *only within* the context of the relationship between 1 and 3.[12]

Example 4.9 (Subjective specialization)

Consider the matching economy developed in Examples 4.7 and 4.8. Within the matching 13 both individuals now develop a deepening of their economic relationship. As described in our previous discussion this ultimately leads to a moderate level of trust and the development of subjective specialization; both individuals specialize their production based on the environment of the matching 13 only.

Endogenous specialization under trade

Individual 1 considers the trade opportunities with individual 3 and consequently optimizes her investment in the acquisition of gathering and hunting skills. Hence, given the investment of individual 3 in the acquisition of gathering skills l_3, individual 1 solves the following problem:

$$\max_{0 \leqslant l_1 \leqslant 1} u_1(v_1, m_1) = \sqrt{v_1 \cdot m_1} \tag{8}$$

subject to

$$v_1 = \frac{1}{2}\left[l_1(1+\alpha^1_{13}l_3)\right]^2 + \frac{1}{2}\left[l_3(1+\alpha^3_{13}l_1)\right]^2$$
$$m_1 = \frac{1}{2}\left[(1-l_1)(1+\beta^1_{13}(1-l_3))\right]^2 + \frac{1}{2}\left[(1-l_3)(1+\beta^3_{13}(1-l_1))\right]^2$$

This optimization problem is based on the trade opportunities emerging within the matching 13. It is assumed that both individuals equally divide the gains from trade. In a fully equivalent fashion we can determine the optimization problem of individual 3:

$$\max_{0 \leqslant l_3 \leqslant 1} u_3(v_3, m_3) = \sqrt{v_3 \cdot m_3} \tag{9}$$

[12] For a comprehensive discussion we also refer, e.g., to Yang (2003, Chapter 3.2).

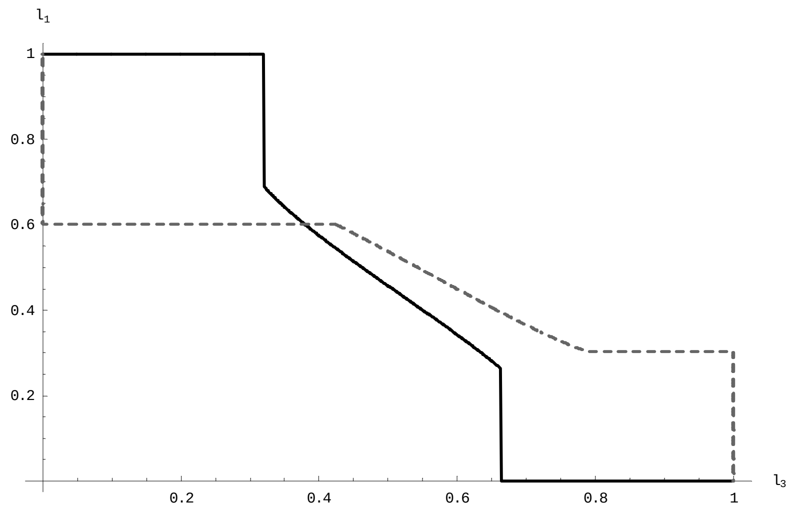

Figure 2: The reaction curves in Example 4.9.

subject to

$$v_3 = \frac{1}{2}\left[l_1(1+\alpha_{13}^1 l_3)\right]^2 + \frac{1}{2}\left[l_3(1+\alpha_{13}^3 l_1)\right]^2$$
$$m_3 = \frac{1}{2}\left[(1-l_1)(1+\beta_{13}^1(1-l_3))\right]^2 + \frac{1}{2}\left[(1-l_3)(1+\beta_{13}^3(1-l_1))\right]^2$$

The reaction functions of players 1 and 3 for the values of the learning parameters given in Example 4.7 are presented in Figure 2. The continuous line represents the optimal investment in gathering skills by player 1 given the investment in gathering skills by player 3 and similarly the dashed line represents the optimal investment in gathering skills for player 3 given the investment of player 1. It is clear that this mutual optimization problem has three solutions, namely the two cases of full specialization: one in which player 1 specializes in gathering and player 3 in hunting and the other in which player 1 specializes in hunting and player 3 in gathering; and an equilibrium of relative specialization in which player 1 specializes relatively more in gathering and player 3 specializes relatively more in hunting.[13] Any of these three solutions indicates a certain level of subjective specialization.

In the two extreme solutions given by $(l_1, l_3) = (1, 0)$ and $(l_1, l_3) = (0, 1)$, in which both individuals fully specialize either gathering or hunting the attained utility levels are $u_1 = u_3 = 0.5$. In the solution of relative specialization given by $(l_1, l_3) = (0.6, 0.4)$ the attained utility levels are $u_1 = u_3 = 0.4344$. Clearly full specialization leads to a higher attainable utility level.

[13]We can reformulate this in game theoretic terms. Indeed, individuals 1 and 3 engage in a two-player normal form game with strategies l_1 and l_3 respectively. The two optimization problems formulate a Nash equilibrium in this game. Thus, we identify three Nash equilibria in pure strategies for this interaction game.

Optimal subjective specialization
We can also compute the Pareto optimal outcome which is given as the solution to the following problem:

$$\max_{0\leqslant l_1\leqslant 1,\,0\leqslant l_3\leqslant 1} \sqrt{\nu_1 \cdot m_1} + \sqrt{\nu_3 \cdot m_3}$$

subject to

$$\nu_1 = \nu_3 = \tfrac{1}{2}\left[\, l_1(1+\alpha^1_{13} l_3)\,\right]^2 + \tfrac{1}{2}\left[\, l_3(1+\alpha^3_{13} l_1)\,\right]^2$$
$$m_1 = m_3 = \tfrac{1}{2}\left[\,(1-l_1)(1+\beta^1_{13}(1-l_3))\,\right]^2 + \tfrac{1}{2}\left[\,(1-l_3)(1+\beta^3_{13}(1-l_1))\,\right]^2.$$

By substituting the given parameter values for α and β, there are two solutions namely the solutions that correspond to full specialization identified by $(l_1, l_3) = (1, 0)$ and $(l_1, l_3) = (0, 1)$. □

5 Objective specialization

The discussion in the previous section clarifies the emergence of stable matching patterns and of subjective specialization. This emergence is essentially based on features *within* the pattern of stable matchings. For an economy to have persistent access to gains from specialization, the social structure of the economy has to *generically* admit stable matchings. Hence, whatever capabilities and desires of the individuals—represented by their utility functions and (possibly) other individualistic features—a stable matching pattern has to exist in the matching economy.

Technically, this brings up the question under which conditions on (N, Γ) there exists a stable matching pattern for *every* possible matching economy (N, Γ, u), where u is an arbitrary utility profile. This line of research follows the research agenda set in the matching literature. Here we are able to apply the main result of Pápai (2004).[14]

Formally we define:

Definition 5.1 *A pair (N, Γ)—consisting of a set of individuals and a potential matching structure on that set—is* ***generically stable*** *if for every utility profile $u \in \mathcal{U}$ it holds that $\Pi^\star(N, \Gamma, u) \neq \varnothing$.*

Our main existence theorem can now be stated as follows:

Theorem 5.2 *The pair (N, Γ) is generically stable if and only if the potential matching structure Γ is odd acyclic.*

Proof.
If: From Definition 4.1 it follows that if Γ is odd acyclic, then $\overline{\Pi}_w$ is odd acyclic too.

[14]Since our analysis is focused on matching patterns, we are able to provide less stringent sufficient condition for stability relative to Pápai (2004).

The sufficiency of odd acyclicity condition on the set of simple matching patterns for the existence of a stable matching pattern follows from Theorem 4.5 directly applied to Definition 5.1. This implies that odd acyclicity of Γ is a sufficient condition for the existence of a stable matching pattern for any utility profile $u \in \mathcal{U}$.

Only if: Suppose that there exists a stable matching pattern for all utility profiles $u \in \mathcal{U}$. Next suppose to the contrary that the potential matching structure Γ is not odd acyclic. Without loss of generality, we may assume that Γ contains a single odd cycle $C = \{i_1, i_2, \dots, i_k\}$ such that $i_1 = i_k$ and $k-1$ is an odd integer.
Take the utility profile $u \in \mathcal{U}$ such that $u_{i_s}(i_s i_{s+1}) > u_{i_s}(i_{s-1} i_s) > u_{i_s}(i_s i_s)$, for all $s = 1, \dots, k-1$ where $i_0 = i_{k-1}$. In every weakly stable matching pattern $\pi \in \Pi_w(u)$ with respect to the utility profile u there is at least one individual, i_s for $s = 1, \dots, k-1$, on the cycle Γ who is relationally autarkic since there are odd number of individuals in the cycle. Thus individual i_s can form a blocking matching with individual i_{s-1} since $u_{i_s}(i_{s-1} i_s) > u_{i_s}(\pi)$, i.e., every individual prefers to be matched with another individual rather than be relationally autarkic, and $u_{i_{s-1}}(i_{s-1} i_s) > u_{I_{s-1}}(\pi)$ since individual i_s is the most preferred partner of individual i_{s-1} given preference profile u. Thus, no stable matching pattern exists in Γ with respect to the given preference profile u.
We now conclude that (N, Γ) cannot be generically stable, which establishes a contradiction. Hence, we have shown the assertion. ∎

Theorem 5.2 provides a complete characterization of generically stable matching structures. This is a very strong result with some deep consequences. Before discussing the consequences of this insight to the discussion of specialization, we turn to the interpretation of the odd acyclicity property.

Theorem 5.3 *Let Γ be a potential matching structure on N. A sub-structure $\Theta \subset \Gamma$ is odd acyclic if and only if (N, Θ) is bipartite in the sense that there exists a partitioning $\{N_1, N_2\}$ of N such that*

$$\Theta \setminus \Gamma_0 \subset N_1 \otimes N_2 = \{\, ij \mid i \in N_1 \text{ and } j \in N_2 \,\}. \tag{10}$$

Proof. It is obvious that every bipartite structure Θ on N is odd acyclic, since all cycles have to be of even length. So, we only have to show the converse.
Let Θ be odd acyclic on N. Without loss of generality we may assume that $\Theta \neq \varnothing$, $\Theta \cap \Gamma_0 = \varnothing$, and that Θ is completely connected in the sense that for all $i, j \in N$ with $i \neq j$ there is a path $P(ij) \subset \Theta$ between i and j.
Select some $i_0 \in N$. Assume that $j \in N$ is such that there exist two distinct paths $P_a = P_a(i_0 j)$ and $P_b = P_b(i_0 j)$ between i_0 and j. We now claim that the length of both P_a as well as P_b are either odd or even. Indeed, if the length of P_a is odd and the length of P_b is even, then $P_a \cup P_b \subset Theta$ defines a cycle from i_0 to i_0 that has an odd length. This violates odd acyclicity of Θ.

Now define $N_1 \subset N$ as follows: For every $j \in N$ we let $j \in N_1$ if and only if the unique length of a path $P(i_0 j)$ is odd. Subsequently we define $N_2 = N \setminus N_1$, consisting of all individuals that have paths of even length with i_0.
Finally, we claim that for any $ij \in \Theta$ it holds that either $i \in N_1$ and $j \in N_2$ or $j \in N_1$ and $i \in N_2$. This follows immediately from the observation that for all $i, j \in N_1$ a path $P(ij)$ between them has to have even length. (Otherwise, there would be an even- as well as an odd-length path between i_0 and i.) Similarly, for all $i, j \in N_2$ a path $P(ij)$ between them has to have even length. ■

Theorem 5.3 states that odd acyclicity of a sub-structure of the potential matching structure Γ is equivalent to this sub-structure being bipartite. The latter refers to familiar structures in matching theory (Roth and Sotomayor 1990) and imposes that relations are only possible between individuals of a different, distinct "type". We develop an interpretation of this requirement in the next sections of this paper.

Our main insight provided in Theorem 5.2 can now be re-stated using the characterization in Theorem 5.3:

Corollary 5.4 *The potential matching structure* (N, Γ) *is generically stable if and only if* (N, Γ) *is bipartite in the sense that there exists a partitioning* $\{N_1, N_2\}$ *of* N *such that*

$$\Gamma \setminus \Gamma_0 \subset N_1 \otimes N_2 = \{\, ij \mid i \in N_1 \text{ and } j \in N_2 \,\}. \tag{11}$$

We now turn to the discussion of the application of this insight to the economies with skill complementarities developed in Examples 3.4, 4.7, 4.8 and 4.9.

As stated before, certain sets of skill complementarities might result into the emergence of stable matching patterns. These stable matching patterns in turn give rise to subjective specialization and mutually beneficial trade. This does not mean that there result widespread gains from trade. For such enhanced economic development it is necessary that there emerges an objective or socially recognized division of labor.

In particular, we argue that the deepening of the stable matching patterns through subjective specialization in turn leads to the emergence of odd acyclic structures of potential matchings. This emergence is based on the social recognition of the roles that are based on the subjective specialization of individuals in such stable matching patterns. This is discussed in the next continuation of Examples 3.4–4.9.

Example 5.5 (Objective specialization)
Consider the stable matching pattern $\pi^* = \{13, 22\}$ discussed extensively in Examples 4.7–4.9 as the unique stable matching pattern. Within this stable matching pattern, the matching 13 is the only binary, value-generating relationship. In Example 4.8 it was sketched that within this relationship there would result trade opportunities if sufficient trust among the individuals 1 and 3 was established. Also, within this matching, individual 1 generated a higher output of meat ($h_1(13) = 0.49$) than of

vegetables ($g_1(13) = 0.39$) and individual 3 generated a higher output of vegetables ($g_3(13) = 0.49$) than of meat ($h_3(13) = 0.39$) due to the actual values of the complementarity parameters α and β.
Subsequently, in Example 4.9 we discussed the emergence of subjective specialization within the matching 13. We identified three different subjective specialization configurations. The emergence of such subjective specialization was based on sufficiently high levels of trust and the presence of a trade relation between individuals 1 and 3.
At present we argue that further deepening of the efficiency in this economy is only possible through the establishment of a true social division of labor. Given the initial output levels, the subjective specialization will develop into the direction as indicated through these output levels. Hence, individual 1 probably specializes subjectively on hunting only, while individual 3 specializes subjectively on gathering only. If these subjective specializations are recognized socially, individual 1 becomes a "hunter" and individual 3 becomes a "gatherer". Being a hunter now becomes a socially recognized economic role, as does being a gatherer. Only after the establishment of these social roles there emerges a *social division of labor*.
Now, the process of objectification of subjective specialization induces the emergence of (social) economic roles in a society. In the example discussed, players 1 and 3 can achieve social recognition as a gatherer and a hunter and re-evaluate their potential utility level from a matching with another player. Now, let player 1 assume the role of a gatherer and player 3 the role of a hunter. Subsequently, assume that there emerge three social roles within this simple economy: **H** stands for a hunter, **G** stands for a gatherer, and **A** stands for an individual in a position of autarky. The assumed skill acquisition of each role is respectively $G_{\mathbf{G}} = H_{\mathbf{H}} = 1$, $H_{\mathbf{G}} = G_{\mathbf{H}} = 0$, and $G_{\mathbf{A}} = H_{\mathbf{A}} = \frac{1}{2}$. The production level of each potential matching is then given by:

ij	$g_i(ij)$	$h_i(ij)$	$g_j(ij)$	$h_j(ij)$
GG	1	0	—	—
GA	1.3225	0	0.64	0.25
GH	1	0	0	1
AA	0.25	0.25	—	—
AH	0.25	0.64	0	1.3225
HH	0	1	—	—

In objective specialization each individual now expects to be trading when she engages in a matching. Also, under objective specialization, unlike under subjective specialization, the level of trust expands to the whole set of players, i.e., to the whole economy. This is why an individual believes fully that she can be matched with another player with whom trade is beneficial in a stable matching. In fact, there is *common knowledge* that gatherers and hunters can be matched in highly productive

social (trade) relationships. Individuals who assume social roles, have socially justified beliefs that a stable matching pattern exists.
In our example, after objective specialization and the establishment of a social division of labor, there emerge three types of individuals: hunters, gatherers and individuals in autarky. A hunter and a gatherer believe that they will exchange half a unit of meat for half a unit of vegetables in a potential matching. The trade between a hunter (or gatherer) and an individual in autarky will take the terms of 0.66125 units of vegetables (meat) for 0.084235 units of meat (vegetables). These are calculated to be the optimal trade patterns in the matchings **AG** and **AH**, respectively.
These production levels now result into the following potential utility levels after trade:

j	**H**	**G**	**A**
$u(\mathbf{H}j)$	0	0.5	0.2360
$u(\mathbf{G}j)$	0.5	0	0.2360
$u(\mathbf{A}j)$	0.4644	0.4644	0.25

.

Clearly, gatherers and hunters prefer to be engaged in a trade relation with each other rather than to be in relation with an individual in autarky. Hence, returning to our three person economy discussed in Examples 3.4–4.9, the unique stable matching pattern can be identified as $\{\mathbf{GH}, \mathbf{AA}\}$, which corresponds to $\{13, 22\}$ in the original setting. □

We argue that objective specialization excludes relationships between individuals with the same social role as being potentially beneficial economic matchings. This implicitly reduces the potential matching structure to an odd acyclic or bipartite structure in which only matchings between individuals with two different roles are recognized.

Finally, Theorem 5.2 does not guarantee the uniqueness of the stable matching pattern that emerges in a matching economy. In order to establish uniqueness we need to impose two additional restrictions on the potential matching structure, namely that $u \in \mathcal{U}_s$ with $\mathcal{U}_s \subset \mathcal{U}$ being the set of all utility representations of strict preferences only and that the potential matching structure Γ is (fully) acyclic, i.e., also cycles with even number of links in the path are not allowed. This result is a direct application of Pápai (2004) uniqueness theorem and, hence, the proof is omitted here.

Proposition 5.6 *Let $\mathcal{U}_s \subset \mathcal{U}$ be the class of utility functions of strict preferences only. Then, the potential matching structure (N, Γ) is generically stable with $|\Pi^*(N, \Gamma, u)| = 1$ for all utility profiles $u \in \mathcal{U}_s$ if and only if Γ is acyclic.*

6 Concluding remarks

In this paper we introduced a four stage approach to the emergence of a social division of labor based on the objective specialization of individuals. As a fifth stage we can add the emergence of market institutions themselves. This approach clarifies that the presence of a social division of labor is in fact a prerequisite for the creation or emergence of a functioning price mechanism. Summarizing these four stages are:

Stage I: Non-equilibrium. In a primitive relational economy without objective specialization, there usually are conditions that do not support an equilibrium. This leads to a situation in which all individuals are trade autarkic and in which there is a state of permanent relational chaos. Individuals are fully self-reliant for the provision of necessities for survival. Consequently, the generated level of welfare is at the level of pure subsistence. Any additional utility generated through interpersonal spillovers from social interaction are purely additional benefits to the generically low subsistence levels. (Example 3.4)

Stage II: Primitive equilibrium. Within a primitive relational economy there might exist conditions that allow the emergence of a stable social interaction pattern. Such a stable pattern is only founded on subjective and personal features, not on any objective or social conditions.

Within this stage we distinguish two sub-stages.

(II-A) At first there only emerges a stable pattern in which interpersonal spillovers are exploited. This first level of stable social interaction facilitates the emergence of a moderate level of subjective trust among the matched individuals. (Example 4.7)

(II-B) Next, the emergence of sufficient subjective trust among the individuals that are engaged with each other, supports the introduction of trade among those individuals; the exploitation of interpersonal spillovers is extended into the trade of economic commodities leading to even higher levels of utilities. The emergence of trade is an important step into the development of an economy. (Example 4.8)

Stage III: Subjective specialization. After trade has been established there is the possibility for a further deepening of interpersonal trust within the stable relationships in the economy. This facilitates the emergence of subjective specialization in which individuals based on the demands of their interpersonal relationships specialize their economic activities. Hence, within the context of a stable trade relationship with other individuals, an individual chooses a production plan to optimize his utility level.

This process of subjective specialization is similar to the specialization process based on inframarginal analysis developed by Yang, e.g., Yang (2001) and Yang

(2003),—as a formalization of the Smith-Young development mechanism—within the context of a perfectly competitive price mechanism. However, subjective specialization does *not* take place within the context of a functioning price mechanism, but rather within the interpersonal relational setting of each individual separately. (Example 4.9)

Stage IV: Objective specialization. The emergence of subjectively specialized individuals can lead to the recognition of social economic roles in the society at large. Individuals who specialize on hunting skills in the context of their individual relationships, become socially recognized as "hunters". Thus, hunters are identified and appointed in the society as producers of meat.

Subsequently, there emerge social rules related to the social role of a hunter as a producer of meat. The engagement of a "hunter" with a "gatherer" in an economically beneficial (trade) relationship may thus become the foundation for economic development. Individuals subsequently specialize in an objective fashion: they now select from a limited set of social roles and engage in an objective fashion with other individuals in their respective social roles to generate mutual economic benefits.

It is only within this context of objective specialization that there emerges a social division of labor which further development acts as an engine for economic growth—described in the context of a market by the Smith-Young mechanism. (Example 5.5)

Stage V: Market emergence. We argue that only after the establishment of a social division of labor based on the social recognition of certain economic roles, there can emerge a functioning market or price mechanism. Besides the social division of labor there have to be established numerous other economic institutions such as the protection of property rights, monetary instruments, and the creation of actual market places. Only after these conditions have been met, there might emerge a price mechanism through which further economic growth and development is made possible in the form of the Smith-Young mechanism based on the extent of the market.

In this paper we only have developed the most basic principles of this descriptive theory. The main conclusion is that economic development and growth is closely related to the development of the social roles in an economy. These social roles have a public nature and as such are subject to a purely public economic theoretical analysis or an evolutionary treatment. This is closely related to the conclusion in Gilles and Diamantaras (2005).

Further development of the abstract theory of matching economies is required before we can expect a full and working understanding of the five-stage process of market development summarized above. This is the objective of future research.

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