The Role of the Medium in an Interaction Structure*

Pieter H.M. Ruys
Department of Econometrics and CentER,
Tilburg University†

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Abstract

In a play, the interaction structure of the roles can be distinguished from the interaction structure of the players on the stage. Both structures have to correspond, whereas the second structure adapts to the first. This distinction is generalized and formalized. A framework is presented to analyse structures of interaction. Roles are described by positions in a projective space. By assuming isomorphy with the dual structure, a unique interaction structure materializes in which some fundamental roles are defined. A correspondence between internal and external interaction is derived. One specific role, the common medium, is singled out. These concepts are identified in the Arrow-Debreu model of a Competitive Equilibrium.

1 Introduction

Interaction is a fundamental feature in real life and in social theory. Although people may interact arbitrarily, interaction is usually institutionalized and occurs between roles. This paper is based on the fact that in a given context the role-structure, i.e., the positions which agents assume, is more stable than the relations between the individual characteristics of these agents. Since many agents may assume a same role, the number of roles is usually smaller than the number of agents. A similar assumption underlies game theory: there are many real life situations in which roles are performed that are analogous to the roles of the prisoners in the Prisoners Dilemma

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Game. The types of interactions in such a situation is determined by the roles between which this interaction occurs. For example, the asymmetric agency relation between a principal and an agent describes a different type of interaction than the symmetric interaction between demand and supply on a competitive market.

The observation that human behavior is determined by the interaction between both situational and personal characteristics is called the interaction paradigm. It is assumed here that the characteristics and the abilities of an agent that plays a role is at least partly determined by that role. When a structure in the roles has been derived, a specification of the ability structure follows. It is the purpose of this paper to design a model that can capture several situations involving interaction between roles and eventually between the characteristics of agents.

One of these types of interaction is strategic interaction between players, fundamental to game theory. There are other types of interaction, which are latent or implicitly present in game theoretical models. This approach gives also a formal structure to a theory called communicative logic, developed by Habermas (1981), which includes the concepts of communicative rationality and goal-rationality. According to Habermas, communicative action is a kind of teleological action, with its own type of goals and its own criteria of rationality, that differs from goal-oriented action. Van Doorne (1982) has shown that this distinction between the various kinds of rationality can remain intact in a structure that simultanuously relates these types of action, using the approach described here.

This claim will be illustrated here by an analysis of the well known neoclassical competitive equilibrium model, also called the Arrow-Debreu model (Debreu, 1959). The organization of the paper is as follows. In Section 2 the two axioms are introduced that constitute the formal model of an interaction context. In Section 3 a meaning is given to the roles that span the interaction context, which leads to the confrontation context. A player of a role is shown to perform in different types of relations. The corresponding different 'faces' of each player will be determined. Section 4 analyses the market context with a.o. the CE-model.

2 The Confrontation Interaction Space

The correspondence between a play and its performance on the stage is used here as a metaphor. A play is a piece of writing to be performed in a theatre describing the acts or the role of each player. A performance is given by a group of players on the stage, each player acting in a specific role. It requires a specific ability of each player to play that role. In the play, the roles are related and interact according to some structure created by the playwright. On the stage the players interact according to a structure, the ability structure, that is supposed to be similar to the role structure.

This analogy is extended to any phenomenon in a real life situation in which roles interact with players. A role may be performed by a player who is natural person, but also an institution or some abstract entity can be a player. A *play* is then a model that describes the *role interaction structure* and the forces that determine the working of some phenomenon. The *performance* of this phenomenon is a set of characteristics associated with the role interaction structure, which constitutes the *ability interaction structure*. These characteristics may have an empirical or an analytical origin, in which case they contain the requirements needed to perform in some role. Both structures together are called the *context* of a specific phenomenon. The role interaction structure of some play may be valid for many specific ability structures. This observation motivates the analysis of roles, set apart from the performance of its players. This property is therefore fundamental for the approach proposed here. But also a player performing in a specific play has usually many more abilities than the one required for a role in that specific play. When these abilities of a player are restrictively specified as necessary for some specific context, the player will be called an agent, characterized by the required attributes. The consistency of the attributes of the various agents in some context is an important object of research. These primitive concepts are used in the following definition.

Definition 2.1 The role and ability structure of a context
The context of a phenomenon consists of two models, $(\Phi, \bar{\Phi})$*. One is the* **role structure***, which identifies the various roles and their interaction and which determines the working of that phenomenon. The other is the* **ability structure***, which specifies the attributes and abilities needed to perform in some role and their interaction and which determines the performance of that phenomenon. A* **player** *of a role carries the abilities needed to perform in that role and interacts with the (abilities of) players of the other roles.*

The role interaction structure of a context is first represented by a graph[1]. A graph, (P, L), consists of a set P of points and a set L of lines between points. A point in the graph represents a role and a line represents a (binary) relation between two roles. A sequence of connected roles in which no role appears more than once is a path. A path with n roles and n lines is an n-fold relation. An n-fold relation is *dependent*[2] if

[1] A **graph**, (P, L) consists of a set of points, $x \in P$, and of a set of lines, denoted by $xy \in L$, each connecting a pair of distinct points in P. A sequence of lines of the form $(x_0x_1, x_1x_2, \ldots, x_{n-1}x_n)$ where all points are distinct except possibly x_0 and x_n, is called a path. A path can therefore also be described by the sequence of points $(x_0, x_1, \ldots, x_n)$. A path is an n*-fold relation* if $x_0 = x_n$ and $n \geq 2$.

[2] The set of interaction relations in the interaction graph (P, C) satisfies the conditions of a superset

each role can be derived from the other roles in the relation. In the sequel, interaction will be defined by an algebraic operation, which requires at least a ternary relation between three roles. An **interaction relation** is a minimal dependent n-fold relation, called a *circuit* of the graph $G = (P, C)$, where P is the set of roles and C the set of circuits. So the the role structure of a phenomenon is represented by the graph G, the **interaction graph** of that phenomenon.

The second step in the analysis is to identify the different types of interaction that are present in the interaction graph G. For that purpose the qualitative role structure described in an interaction graph needs to be embedded in a space in which the different roles and the different interaction relations can be identified and located. This step is comparable to representing a geographic territory with cities and a road system by a cartographic map. Such a map is embedded in the 2-dimensional Euclidean space in which a distance is suitably defined. The Euclidean space, which describes interactions as addition and multiplication, is suited for a world in which a zero-element exists that allows for reversible operations and operations on identical elements. This is needed in a mechanistic and probabilistic world. But that Euclidean space is not an appropriate tool to represent an interaction context. For, the addition of two identical distances has sense in a territory, but interaction between two identical roles in a context has no sense and is not defined. The geometric tool that serves our purpose it the projective geometry[3].

It is therefore assumed that the interaction graph G can be represented by a projective space $G_n = (P_n, \oplus)$, called the **interaction space** of the interaction structure. In the interaction space each role assumes a unique position in P_n, so each role can be *located* in a projective space. The interaction between roles is defined by the binary operation $\oplus$, which assigns to two distinct roles a unique third role in the space and

system. A superset system is a pair (L, R), where R is a family of dependent sets –or interaction relations – in L having the property that if some set A belongs to R, then a set B containing set A also belongs to R. An inclusion-wise minimal dependent set is called a circuit. A set that is not dependent is called independent. An inclusion-wise maximal independent set is called a base. A superset system in which any two bases have the same number of elements is called a matroid.

[3] A projective geometry PG(n,2) is an n-dimensional **projective space** $(P_n, \oplus)$ over the field with two elements, $\{0, 1\}$. The projective space consists of a set of positions P_n on which a binary operation $\oplus$ is defined. Each **position** in that space is a vector of $n + 1$ coordinates; each coordinate is either 1 or 0, but not all are 0. The operation $\oplus$ assigns to any pair of distinct vectors a unique third vector in the space by the following rule: $0 \oplus 0 = 1 \oplus 1 = 0$ and $0 \oplus 1 = 1 \oplus 0 = 1$ for each coordinate. The operation is called coordinate-wise addition modulo 2. The number of positions is equal to $(2^{n+1} - 1)$.

With each position $x = (x_0, x_1, \ldots, x_n)$ in P_n a unique subset $H(x) = \{p^0, p^1, \ldots, p^n\}$ can be associated, where $p^i = (p^i_0, p^i_1, \ldots, p^i_n)$, that consists of the $n + 1$ solutions of the equation: $x_0 p_0 \oplus x_1 p_1 \oplus \ldots \oplus x_n p_n = 0$. This subset is called a **hyperspace**. The map $H : P_n \to P^*_n$ is called the *hyperspace map*. A 1-dimensional subspace is spanned by two independent positions, say x and y, and is equal to the set $\{x, y, x \oplus y\}$ in P_n. The number of these 1-dimensional subspaces is equal to $(2^{n+1} - 1)(2^n - 1)/3$. A ternary relation is a 1-dimensional subspace.

thus a ternary relation. This operator can adequately represent the concept of dependence and of interaction introduced above. This implies that also types of interaction can be identified and located in the geometric space. So the first step is achieved by the following axiom.

Axiom 2.2 Representability
The interaction structure of a context is represented by the interaction space in a projective geometry.

The representability axiom provides the possibility to identify the roles and their interaction in an interaction structure. The representation of interaction by a projective geometry is the most general and subtle one for describing the working of interaction within a phenomenon. This benefit goes at a cost, of course. The cost is that the interaction operation is a binary operation. So it is assumed that any multiple, n-fold interaction relation can be decomposed into binary interactions. The basic building block of interaction is a ternary relation, which implies a form of separability in interaction. A second type of cost is that a position is identified by a vector in the projective space. A vector consists of an ordered number of *components*, where each component is usually a real number that receives a meaning from the model in which the vector is defined. In the case of a projective geometry the number of components equals $n + 1$, where n is a finite nonnegative number, called the *dimension* of the projective space. Each component is either the number 1, or the number 0, so it can express only whether some quality relevant for the context is present or not. The representability axiom implies that a composition of these qualities, henceforth called *capabilities*, gives a full description of the abilities of a player who performs a specific role in a context.

Proposition 2.3 Identification of each role
Let a role interaction structure of a context satisfy the representability Axiom 2.2. Then any role in the interaction structure or in the interaction graph $G = (P, C)$ *is uniquely represented by a* **position** *in some* n*-dimensional projective space, the interaction space* $G_n = (P_n, \oplus)$*. This position is a vector composed of* $n+1$ *numbers zero or one, indicating whether some of the* $n+1$ *capabilities in the context is absent or present in that position.*

The projective geometry in which the interaction structure[4] is embedded allows also the identification of an interaction relation or circuits in the set C of the interac-

[4] The interaction structure embedded in a projective geometry is a binary matroid, which is a matroid that is representable over the projective geometry PG(n,2). The superset structure of interaction relations in the interaction graph defines a minimal dependent set of this matroid. The set of minimal dependent sets is closed under the operation of symmetric differences.

tion graph (P, C). For, an interaction relation is identified by a hyperspace and with each hyperspace a position in the space $(P_n, \oplus)$ is associated. This follows from the duality property of a projective space. Next interaction is identified.

Proposition 2.4 Identification of interaction
Let a role interaction structure of a context satisfy the representability Axiom 2.2. Then any interaction relation in the interaction graph $G = (P, C)$ *is uniquely represented by a* **hyperspace***, an* $(n-1)$*-dimensional set in the space* $(P_n, \oplus)$*. The space* $(P_n, \oplus)$ *is equivalently described by the graph* $G_n = (P_n, C_n)$*, where* $P_n = P$ *and the set of hyperspaces* $C_n = C$*. So* $G = G_n$*. Each hyperspace corresponds with a unique position in the space and is an interaction relation, i.e., if for some* $p \in P_n$*,* $H(p) = \{a, b, \ldots, f\}$*, then* $a = b \oplus c \oplus \ldots \oplus f$.

The role interaction structure and its associated ability structure may generate another context of the same phenomenon, the second level context. The second level role interaction space G^*_n has the hyperspaces defined in the first level space as elements that assume role-positions. On these role-positions an interaction operations is defined. The hyperspace map assigns to each element in this space a unique position and defines[5] the interaction operation $\oplus^*$. The interaction $\oplus$ between roles on the first level also determines the interaction $\oplus^*$ between roles on the second level. These form together the second level interaction space $(P^*_n, \oplus^*)$, where $P^*_n = C_n$. This interaction space $(P^*_n, \oplus^*)$ can equivalently be represented by the role interaction graph (P^*_n, C^*_n), where $C^*_n = \{H(a) | \exists A \in C_n : a \in A\}$ and $P^*_n = P_n$.

Definition 2.5 The second level role interaction space
Let a role interaction space $(P_n, \oplus) = (P_n, C_n)$ *be given. Its* **second level role interaction space** $G^*_n = (P^*_n, \oplus^*)$ *consists of the set* P^*_n *of positions that are associated with hyperspaces in the first level space and a corresponding interaction operation* $\oplus^*$*, i.e., if, for* $\{a, b, \ldots, f\} \in C_n$*,* $a = b \oplus c \oplus \ldots \oplus f$*, then* $H(a) = H(b) \oplus^* H(c) \oplus^* \ldots \oplus^* H(f)$.

The ability structure on the second level consists of a set of abilities together with an interaction operation between these abilities. Each ability A in $\bar{C_n}$ is itself an interaction relation, determined on the first level: $A = (a = b \oplus c \oplus \ldots \oplus f)$. This interaction is called internal interaction within a positional ability. The interaction between these positional abilities is called external interaction on the second level.

[5] The interaction operation $\oplus^*$ in the dual space P^*_n is defined by the hyperspace map $H : P_n \to C_n$, where $H(a \oplus b) = H(a) \oplus^* H(b)$ for all elements a and b in P_n.

Definition 2.6 The second level ability interaction graph
Let a role interaction space $(P_n, \oplus) = (P_n, C_n)$ *be given. Its second level ability interaction graph* $\bar{G}^*_n = (\bar{C}_n, \bar{C}^*_n)$ *consists of two sets: the set* $\bar{C}_n$ *of abilities determined by a specific* **internal interaction***,* $\oplus$*, and the set* $\bar{C}^*_n$ *of interaction relations determined by the* **external interaction***,* $\oplus^*$*. So for abilities* $A, B, \ldots, F$ *in* $\bar{C}_n$ *on the second level, if* $\{A, B, \ldots, F\}$ *is a minimal dependent set in* $\bar{C}^*_n$*, then* $A = B \oplus^* C \oplus^* \ldots \oplus^* F$.

The question is whether these abilities can be subjected to the interaction rules $\oplus^*$ of the role structure, viz., whether two abilities can assign a third one just as two roles assign a third one. These roles are separable, so a minimal dependent relation may consists of more than three roles. This is not necessarily the case for abilities. In fact, the minimal interaction relation as a derived ability allows only a ternary relation to meet the separability requirements. For a dimension greater than two, however, this third position is not a minimal interaction relation on the first level and therefore excludes the representation of this structure in a projective space. By requiring that the second level structure is also be representable in the projective space and isomorphic[6] to the role structure, a unique interaction structure follows.

So, on the first level no formal difference has been made between the role structure and the ability structure. On the second level, the role structure is by definition equal to the first level role structure. The ability structure depends on the question whether internal interaction is consistent with external interaction, or $(\bar{C}_n)^* = \bar{C}^*_n$. The isomorphy axiom implicitly also applies to the case of a play introduced above: there exists an isomorphy between the roles in a play and the players, i.e., the required abilities, on the stage.

Axiom 2.7 Isomorphy
Let a context of a phenomenon be given. There exists an isomorphy between the role interaction space and the ability interaction space in the context of a phenomenon.

The representation axiom implies that the only dimension for which the dual graph G^*_n is representable by a projective space is the dimension 2. The isomorphy axiom[7] implies that the number of positional abilities in the second level structure equals the number of ternary interaction relations. So the interaction space is restricted to a 2-dimensional space. This is expressed in Proposition 2.8.

[6]Two algebraic structures $(P, \oplus)$ and $(R, \oplus)$ are **isomorphic** if there exists a one-to-one mapping g from P in R such that $g(a \oplus b) = g(a) \oplus g(b)$ for all elements a and b in P.

[7]This axiom implies that a role derives its meaning in that it represents an ability, which is a possible state of the world. It therefore belongs to the representation theory of propositions. The isomorphy axiom implies that the representation *mapping* is generalized to an *interaction relation* between a role structure and an ability structure.

Proposition 2.8 *Let the context of a phenomenon and its corresponding second level context satisfy the representation Axiom 2.2 and the isomorphy Axiom 2.7. Then the dimension n of the interaction space is equal to 2.*

The 2-dimensional interaction space mentioned in Proposition 2.8 is called **the tripolar interaction space**. It is denoted by G and depicted in the diagram in Figure 1. It allows for a modular approach and enables the comparability of contexts for a large variety of phenomena. This is of great practical value. Since most phenomena have different levels of interaction, it can be used as a microscope or a teloscope under some conditions to be specified later.

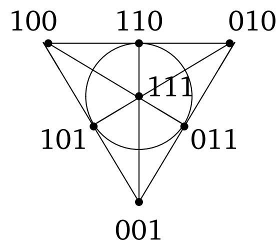

Figure 1: The tripolar interaction space

The tripolar interaction space[8] has 7 positions each identified by a vector with 3 coordinates having a value 1 or 0, which means that some characteristic is present or absent [9]. Any two positions in this structure determine a minimal interaction relation, i.e., a line or circle in the diagram. For example, 100 and 010 determine uniquely the position 110 through the interaction operation $100 \oplus 010 = 110$, and thus determine the relation {100, 010, 110}. Any position in this relation can be derived from the other two, e.g., $010 = 100 \oplus 110$, these three positions are called interactive or dependent. On the other hand, the positions 100, 010 and 001, e.g., do not interact directly and are called independent because $100 \neq 010 \oplus 001$.

The three polar positions are independent. When the positional abilities on these

[8]This structure, which is a matroid representable over the projective geometry PG(2,2), is also known as the Fano plane or the Fano matroid. There are 35 different combinations of three positions in G, 7 of which are minimal dependent subsets or circuits in the set G and constitute the set G^*. The 28 other sets of three positions are independent in G. Any independent set of three positions spans the whole structure.

[9] One of these positions is unique, 111, containing all 3 characteristics. The three positions that contain 2 characteristics are symmetric, as well as the three positions containing 1 characteristic.There are also 7 relations. A unique one where the 3 member positions contain 2 characteristics (the circle), three symmetric relations that each contain 2 positions with 1 characteristic, and three symmetric relations that each contain 1 positions with 3 characteristics.

three positions have been specified, or, equivalently, the capabilities on each of the three coordinates, the whole interaction context is reconstructed. A polar position is therefore called a **spanning position**, both in the role space and the ability space. By determining these spanning positions, or equivalently the three spanning capabilities, a context is specified. The generic context in this paper is called the confrontation context. It is determined as follows. A context of a phenomenon is constructed by specifying the spanning capabilities of the ability context. The following non-formal argument leads to such a specification. If all seven positional abilities would have nothing in common, then no confrontation can take place and no interaction can occur. On the other hand, if there is no difference between positional abilities, then there is nothing to interact. So two of the three spanning capabilities should be symmetric and be given an opposite meaning that incites interaction, and one spanning capability should facilitate interaction by means of making a confrontation between opposite meanings possible. This reasoning leads to a *generic interpretation* of the three polar positional abilities, corresponding with the three spanning capabilities, as is expressed in the following definition.

Definition 2.9 The confrontation interaction space
A tripolar interaction space is a **confrontation space** *if the three spanning capabilities have the following generic meaning:*

1. *the capability of two positions each being* **opposite** *towards the other position;*

2. *the capability of the third position of being a* **common medium** *that facilitates confrontation between the two opposite positions.*

Both the role interaction space and the ability interaction space are isomorphic to the confrontation space. The confrontation space is assumed to be generic description of the role interaction space for all phenomena presented. The ability interaction space is specified for each occasion, but in correspondence with the role interaction space.

3 Roles and abilities in the confrontation space

When the working of a phenomenon is analysed on one level only, it is only required that positional-abilities fit the role-positions in which they have to perform. In this section the most general description of all roles in the confrontation space is given, which description is determined by (i) the spanning capabilities defined in Definition 2.9 and (ii) the interaction operation of the confrontation space. The meaning of the roles determined by interaction structure prevails therefore over the descriptions given below. For the location of the positions in the context is referred to Figure 2.

The generic meaning of roles in the confrontation space

polar opposites, a, b These spanning positions are defined in Definition 2.9 as carrying the ability of being symmetrically opposite towards each other. They are located at the points 100 and 010 in Figure 1, and at the points a and b, or A and B in Figure 2.

medium, c This spanning position carries the ability of transforming a polar opposite ability into an actual opposite ability. Its role is thus to enable some polar ability to enter into a confrontation with another opposite ability. It is located at point 001 in Figure 1, and at positions c, or C, in Figure 2.

actual opposites, d, e These positions carry the ability of a confrontation between opposite abilities, which is needed for the communication. These positions are located at the positions 101 and 011 in Figure 1, and the positions d and e, or D and E in Figure 2.

common position, f The common position carries the ability of a meeting of opposites. It may be considered as the endogenously determined goal of the interaction structure. This role is located at position 110 in Figure 1, and position f, or F, in Figure 2.

contextual identity, g The identity position carries the ability of integrating all capabilities. This role is located at position 111 in Figure 1, and at position g, or G, in Figure 2.

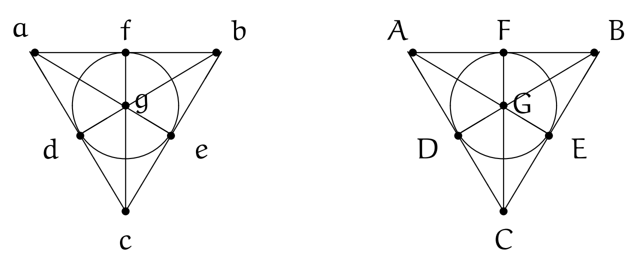

Figure 2: The first and second level confrontation space

To following example may help to support the intuition. Consider a discussion between two persons. For describing the working of this phenomenon, i.e., the interaction between the roles, some specification of abilities associated is chosen such that it fits the confrontation role space of a discussion. This space is spanned by the polar opposite positions, a, b, which carry the two opposite opinions of both participants and the polar medium position, c. The medium carries the ability of each of both participants to express his or her opinion to the other. One may think of a common language or any other communication device. Using this tool, both participants are able to confront each other with the other discussant's opinion, represented by the positions d, e. This confrontation leads, in case of interaction, to a common understanding carried by the common position, f. The communicated common understanding is carried by the contextual identity, g.

Next the various interaction relations are identified.[10] The locations of the positions with their corresponding type of internal interaction are depicted in Figure 2.

The generic meaning of interaction relations in the confrontation space

polar-incorporation interaction, $\{\mathbf{a}, \mathbf{d}, \mathbf{c}\}$ or $\{\mathbf{b}, \mathbf{e}, \mathbf{c}\}$ This type of interaction enables a polar-opposite ability to be incorporated by means of the medium ability into an opposite ability which can engage in contextual confrontation.

communication, $\{\mathbf{a}, \mathbf{f}, \mathbf{b}\}$ The communication type of interaction describes the interaction between polar-opposite abilities and the common position.

instrumental, $\{\mathbf{a}, \mathbf{g}, \mathbf{e}\}$ or $\{\mathbf{b}, \mathbf{g}, \mathbf{d}\}$ The instrumental interaction refers to the type of interaction between a polar-opposite ability and its actual-opposite ability.

association, $\{\mathbf{f}, \mathbf{g}, \mathbf{c}\}$ The association type of interaction refers to the integration of the contextual goal carried by the common position and its means carried by the medium position into the contextual identity.

confrontation, $\{\mathbf{d}, \mathbf{f}, \mathbf{e}\}$ The confrontation type of interaction describes the interaction between actual-opposite capabilities and represents the actual adaptation process of the opposite capabilities, eventually attaining This type of interaction gives the name to the whole reconstructed context.

[10] The following convention is accepted for the *notation* of a relation. A minimal interaction relation in the graph G is denoted equivalently by the three lines between positions, e.g., $\{ad, dc, ca\}$, or by the three positions, $\{a, d, c\}$.

The capabilities of positions and the types of interaction in the confrontation context are summarized in the following table. The first and second level confrontation space are denoted by resp. CS1 and CS2. The second level space has been introduced in Definition 2.5. The position which each type of interaction on the first level assumes a position on the second level is defined by the hyperspace map defined in Definition 2.3.

CS1		CS1		CS2
pos'n	role	relation	type of interaction	position
a, b	polar opposite	$\{a, d, c\}, \{b, e, c\}$	polar incorporation	A, B
c	medium	$\{a, f, b\}$	communication	C
d, e	opposite	$\{a, g, e\}, \{b, g, d\}$	instrumental	D, E
f	common position	$\{f, g, c\}$	association	F
g	contextual identity	$\{d, e, f\}$	confrontation	G

Above, each ability has been related to a position, the **positional ability** has been determined. Each such ability can be differentiated in the confrontation space according to the interaction relation in which that ability performs. So each player in a confrontation space may be considered to have three 'faces', one in each direction of an interaction relation.

4 Relational abilities in the context of a game

In the previous section, the positional abilities of players in a confrontation space have been derived. Since each player is engaged in interaction in three distinct interaction relations, he may have three different perspectives or orientations in one and the same role or position in the confrontation space. Such a differentiated positional ability is called a **relational ability**, associated with some position. These relational abilities are needed to describe and analyse specific models by means of the confrontation space. The first such model is that of a game.

A strategic game is a model of interactive decision-making in which each decision-maker chooses his plan of action once and for all, and these choices are made simultaneously. A model of a game with two players, a and b, consists of a set of actions for both players, X^a, X^b, a set of consequences Y of a choice of action by both players and a function that associates with each pair of actions a consequence for each player, $g : X^a \times X^b \to C$. Each player has a preference relation $\preceq^a, \preceq^b$ on the set of consequences. Under a wide range of circumstances the preference relation $\preceq^a$ of player A can be represented by a payoff function $U^a : C \to R$, also called a utility function, in the sense that $U^a(x_1) \geq U^a(x_2)$ whenever $x_1 \succeq^a x_2$.

The confrontation sapce of such a game is spanned by the polar positions. Two opposite abilities specify the ability of each player to assess the consequences of actions, and implicitly the actions itself. The medium position carries (i) the ability to relate an action profile with a consequence profile, and (ii) the common rules of the game, viz., the way information is exchanged and the equilibrium concept. The governing interaction structure of the context specifies the abilities on the other positions. The results are described in the following table. It may be noticed that all symmetric positions in the context are represented in the table by one of the two, i.e., a may be substituted for b, and d may be substituted for e. The same holds for the relations.

CS1 pos'n		relational ability	CS1 relation	type of interaction
a, b	a_C	preferences on consequences	$C = \{a, f, b\}$	communicating
	a_A	preferences on own actions	$A = \{a, d, c\}$	incorporating
	a_D	preferences on other actions	$D = \{a, g, e\}$	instrumental
c	c_A	relating conseq. with actions	$A = \{c, d, a\}$	incorporating
	c_F	common rules	$F = \{c, g, f\}$	integrating
d, e	d_A	actions from own consequences	$A = \{d, a, c\}$	incorporating
	d_G	actions from other actions	$G = \{d, e, f\}$	confronting
	d_E	actions from other consequences	$E = \{d, g, b\}$	instrumental
f	f_C	equilibrium in consequences	$C = \{f, a, b\}$	communicating
	f_G	equilibrium in actions (Nash)	$G = \{f, d, e\}$	confronting
	f_F	equilibrium signals from rules	$F = \{f, g, c\}$	integrating
g	g_D	reference outcome	$D = \{g, a, e\}$	instrumental
	g_F	game outcome	$F = \{g, f, c\}$	integrating

So the positional abilities of the medium position are twofold: one is oriented at the polar-opposites to transform opposite interests into opposite actions that enable a common outcome, denoted by c_A in the table; the other is oriented at the common position to transform the common equilibrium concept into a common outcome of the game, denoted by c_F. Since these polar agents are all given and fixed in the game context, the only dynamics that is left in this formal description of a game is an adaptation process. Such a process takes place in the confrontation relation.

In order to extend the scope of the confrontation space to models with more than two players in an opposite but symmetric role, the concept of congruence[11] is introduced. The essential idea of symmetric (polar-)opposites is that both their abilities differ and are complementary in the sense that the one needs the other to develop a common profile. In principle, two is the smallest number to comply with this requirement. A larger number of agents is feasible whenever the property of being opposite survives if the model is restricted to any pair of agents. This is the case if agents are distinct and congruent, i.e., the specifications of an agent meet the essential properties required by its position. The is expressed in the following definition.

Definition 4.1 congruence
A player that is endowed only with positional abilities that are needed in the model to perform in some role of a confrontation space is an **agent** *in that position of the model. Two agents in a model are* **congruent** *if they have different attributes, but can perform in the same role or in an opposite role of the confrontation space.*

The effect of congruence for the various relations in the confrontation space is that the incorporation relations can be aggregated to an incorporation domain. In other relations the fundamental opposite capability between two players in Definition 2.9 is extended to capabilities describing **variability** between many players in a domain. Domains are suited to describe competition. This result is presented in the following table.

Types of domains in the confrontation context

communication domain This social domain extends the communication relation **C** between two abilities to a communication system between a set of abilities.

confrontation domain This social domain extends the confrontation relation **G** between two abilities to a confrontation system between aggregated abilities.

association relation The association relation **F** relates the communication and the confrontation domains and results from the instrumental domain.

incorporation domain This is the domain of individual incorporation relations **A**, **B**.

[11] In geometry congruent figures have identical geometric properties. Two figures are called congruent whenever corresponding Euclidean measurements in the two figures always agree: corresponding sides have the same length, corresponding angles have the same measure, and so forth.

instrumental domain This domain contains all the instrumental relations **D**, **E**. It forms a complex network of strategic interacting relations, actions and reactions, that may stabilize the interaction structure governing the context.

The instrumental domain belongs to the core of game theory. Strong assumptions are required to describe it precisely. The medium-role occurs in the incorporation domain and the association relation. Its relational ability is indicated by the following definition.

Definition 4.2 the revelation property
If an ability in some interaction relation of a confrontation space can technically be revealed from the other two abilities in that interaction relation, this revelation is **deduction** *if the ability is deduced from two polar abilities in the interaction relation. It is* **induction** *if it reveals a polar ability.*

In economic models the derivation of individual preferences from individual behaviour is called revelation. An example is a utility maximizing consumer in a competitive market. The two given positional abilities are the individual preferences and resources on the one hand and the market opportunities on the other hand. The consumer choice can be deduced. A reverse direction is when the set of choices under various market opportunities is given and polar positional ability of the individual preferences results. This is 'revelation' of preferences is induction of an opposite polar ability. The third direction is induction of the medium role, as presented by Kaneko and Matsui (1999). They induce the common rules from behavior and preferences. Ruys and Storcken (1997) have derived conditions on the (common) decision rule for individual rational choice.

5 The confrontation space of a market economy

There are many contexts in which the economic phenomenon of a market can be described. The context presented here is the neoclassical model of a market economy as described by Arrow and Debreu (1954). In the table below the lowercase letters indicate abilities, the uppercase letters relations in Figure 2.

TRIP pos'n		relational ability	TRIP relation	type of interaction
a, b	a_C	nil	$C = \{a, f, b\}$	nil
	a_A	preferences on commodities	$A = \{a, d, c\}$	incorporating
	a_D	nil	$D = \{a, g, e\}$	nil
c	c_A	allocation of resources	$A = \{c, d, a\}$	incorporating
	c_F	market rules	$F = \{c, g, f\}$	asociating
d, e	d_A	individual choices	$A = \{d, a, c\}$	incorporating
	d_G	exchange transactions	$G = \{d, e, f\}$	confronting
	d_E	nil	$E = \{d, g, b\}$	nil
f	f_C	nil	$C = \{f, a, b\}$	nil
	f_G	equilibrium choices (Nash)	$G = \{f, d, e\}$	confronting
	f_F	equilibrium prices	$F = \{f, g, c\}$	associating
g	g_D	nil	$D = \{g, a, e\}$	nil
	g_F	outcome	$F = \{g, f, c\}$	associating

The incorporation domain contains the relational abilities needed for a trader in the market to express a rational choice. The 'preferences on commodities', $a_A = a_{(a,d,c)}$, stands for the description of the characteristics of traders in the economy, viz., their individual resources and preferences on commodity bundles. The 'allocation of resources',$c_A = c_{(c,d,a)}$, stands for the conditions of the allocation mechanism relevant for the individual traders, such as the valuation and allocation of individual resources in accordance with the market rules. The rational choices of traders, d_A, result from optimizing behavior in this relation. In the association domain, the relational ability $c_F = c_{(c,g,f)}$ represents the effects of the 'market rules' common to all traders are, such as the private property conditions, the optimizing behavior of traders, and the prices communicated to all traders. These market rules interact with f_F, the 'equilibrium prices', resulting in g_F. The confrontation domain, 'equilibrium choices' are made when no trader can improve upon the allocation, given the equilibrium prices, whereas 'equilibrium prices' clear all markets in the economy, both together satisfying the conditions of a Nash-equilibrium.

The communication domain between traders and the instrumental domain are not developed in this market confrontation space. From this description of the an Arrow-Debreu economy it follows that the interaction structure in this model is very restricted. The only type of dynamics in the Arrow-Debreu framework is the price adaptation process. It also shows how important the role of the medium is in the Arrow-Debreu market context. It is remarkable that the paradigm of methodological individualism (Schumpeter 1908) has been based on this model, whereas the confrontation space reveals that role of the medium is predominant and the individual components play only a role in one type of interaction (incorporation). Although game theory has filled this gap for another type of interaction (instrumental), the role of the common medium has never since decreased.

The **medium** operator facilitates the transformation of a complex structure from one relation (the communication domain) into another (the confrontation domain), leaving the essential characteristics of the structure invariant. Focussing on the passive role of the medium to transform a given structure, the *power* of the medium ability is determined by (i) the degree of complexity of the structure, and (ii) by the degree of invariance obtained under the transformation. A medium is *effective* if it is able to transform the structure in one domain within a given degree of invariance into a structure in the other domain. It is *efficient* with respect to the communication domain if a coarser medium-operator is not able to transform a structure within the given degree of invariance. The power (effectiveness) and the performance (efficiency) of the medium is thus determined by the communication domain of the confrontation space.

The required abilities of the medium role in a confrontation space may vary from extremely complex to extremely simple. Formal models are usually endowed with a simple medium ability, such the duality operator in the confrontation space of the linear programming model. When suitable conditions on the polar abilities are satisfied, a simple duality operator transforms perfectly the production structure from the quantity space (the confrontation relation) into the price space (the communication relation). However, since this medium performs the transformation process 'perfectly' in the linear programming confrontation space, it does not add anything to the production structure: it does not interact with this structure in the sense developed here. This in contrast to real life phenomena where the medium in a confrontation space influences the outcome considerably. But also in contrast to more sophisticated specifications of the medium role in formal models, where allocation mechanisms and mechanism design aim at describing the individual specifications in the communication domain as efficient as possible to arrive at solutions in the confrontation space that are as close as possible to real life situations. The Arrow-Debreu model is just a start in this development.

6 Conclusion

Auman (1985) alluded to a medium with a complex capability when he referred to the relation between game theory and art: "A characterization of art that I find very apt is 'expression through a difficult (or resistive) medium'. ... The resistiveness of the medium imposes a kind of discipline that enables – or perhaps forces – the artist to think carefully about what he wants to express, and then to make a clear, fortright statement. In game theory and mathematical economics, the restrictive medium is the mathematical model with its definitions, axioms, theorems and proofs." The mathematical model – the formal language – is the medium that transforms an envisaged structure into a mathematical structure, leaving the essentials of the structure invariant.

In social choice theory the medium role performs as an aggregator of individual preferences. The confrontation context approach may help to analyse the performance of this common medium, both as a formal operator and as a specific function or role in an actual social play.

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