Optimal Provision of Infrastructure Using Public-Private Partnership Contracts[1]

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Abstract

This paper contributes to the design of an infrastructure charging framework. Its aim is to design rules for establishing and operating an infrastructure service, which supports an allocative efficient solution that is normally budget neutral. An infrastructure service is a service that can be decomposed in at least two service levels. The first-level services represent the private use that is made of the second-level public service. The first-level services are marketable, but cannot be used without the complementary service on the second level. This infrastructure service concept has an analytical dimension, which allows to derive efficiency and existence properties, and a governance dimension, which allows for applying transaction cost theory in choosing an optimal policy. This governance is characterized by Public-Private Partnership contracts, which are principal-agent relations in which the public authority sets specific rules that firms in the private domain are able and willing to accept in order to provide both the first and the second-level infrastructure service. So the public authority establishes an optimal infrastructure service by providing rules rather then productive resources.

Key words: infrastructure, public goods, public-private partnership, governance, efficiency, general equilibrium.

1 Introduction

The governance of an infrastructure service features high on the agenda of both economists and politicians. It is clear that satisfactory solutions have not been found yet. For example, the state of California has recently issued a “stage-three” emergency for electricity provision, meaning that 98.5% of its power reserves had been consumed and that a series of hour-long power cuts might be imposed on different regions. The underlying problem is that, although California's demand for energy has risen rapidly in the past decade of boom, it has built very little new generating capacity[1]. Another example is the vivid discussion about the restructuring and privatisation of utilities, see [15], and about the boundary between the public and private domain. The European Union has initiated a discussion on how to protect services of general interest in a competitive market environment. More specifically, the European Commission has published recently a White Paper [12] on a common transport infrastructure charging framework.

This paper contributes to the design of an infrastructure charging framework. Its aim is to design rules for establishing and operating an infrastructure service, which supports an allocative efficient solution that is normally budget-neutral. An infrastructure service is a service that can be decomposed in at least two service-levels. The first-level services represent the private use that is made of the second-level public service. The first-level services are marketable, or can be made marketable, and belong to the private domain ruled by competitive markets. These private services, however, cannot be used without the complementary service on the second level. The second-level service is a nonmarketable service and may belong to the public domain. It is a local public good or a network with controlled access. This infrastructure service concept has an analytical dimension, which allows to derive efficiency and existence properties, and a governance dimension, which allows for applying transaction costs theory in choosing an optimal economic policy. This governance dimension is characterized by Public-Private Partnership contracts, which are principal-agent relations in which the public authority sets specific rules that firms in the private domain are able and willing to accept in order to provide both the first and the second level infrastructure service[2]. So the public authority provides in principle productive rules rather than productive resources in order to obtain an optimal infrastructure service.

The problem of providing and financing infrastructure services has a long history. The role of the public authority has changed during the last two centuries. We distinguish

[1] See *The Economist* December 23rd 2000.

[2] The Dutch Ministry of Finance describes PPP as "a partnership in which the government and the private sector together carry out a project on the basis of an agreed division of tasks and risks, each party retaining its own identity and responsibilities." This description implies detailed government involvement, which may be practically indispensable for a learning by doing procedure at the start of such an enterprise.

three degrees of government involvement. The first degree is characterized by a fully public provision of the entire infrastructure service. This service may be organized as a vertically integrated utility. The public authority determines the content of the service and executes this task. In the second degree of involvement, the public authority sets the content of the infrastructure service and defines separate tasks on the different service-levels and lets private operators execute some of these tasks. In the third degree of government involvement, the public authority only sets the governance of the infrastructure service, which governance supports the underlying economic structure. This governance consists of defining principal-agent relations between complementary level services in such a way that the downstream agents, who render first-level services directly to the public, can perform their tasks optimally. This implies that the stream of information and delegation between levels goes up and down, in two directions. This solution concept is particularly relevant if a supernational government is absent. But it presupposes a system of good performing institutions on each service-level and a system of enforceable agency contracts. So the role of the public authority shifts from determining the content of its service to determining and controlling the governance of its service. This shift in role increases the efficiency of the provision of an infrastructure service, as well as improves the position of equity considerations. For the care of equity aspects can be inserted in the most appropriate level of the governance.

The main economic issue to be solved in the case of first-degree government involvement is public utility pricing. The French engineer Dupuit [7] proposed already in 1844 the marginal cost pricing rule for utilities in order to optimize the welfare effects of an integrated public utility. This rule implied budget losses to be financed by lump-sum taxation. That insight has not changed in 150 years, see Cornet [4]. The case of balanced budgets was considered by Boiteux and Ramsey, leading to the Ramsey-Boiteux pricing rule: the mark-up on marginal cost to meet the breakeven constraint should be inversely proportional to the price elasticity for the different products sold by the utility. This rule is budget neutral but not efficient. Brown, Heller and Starr [2] define a two-part tariff pricing rule that recovers the losses incurred by pricing at marginal cost by a hookup charge for access to any purchases of the monopoly good. The hookup charge is a fixed charge that is imposed on any buyer wishing to purchase any positive amount of the increasing returns good. It may be uniform across buyers or it may vary across buyers. The variable charge consists of a constant per unit charge equal to the marginal cost of production. They show the existence of such a two-part marginal cost pricing equilibrium, where only households are required to pay an access charge. Such an equilibrium does not need to be efficient. Moreover, some Pareto efficient allocations cannot be supported as two-part tariff equilibria. See also Hammond and Vilar [14] and Vohra [24].

The second degree of government involvement implies unbundling of the vertically integrated industry. Unbundling allows for separate pricing at each level. The second-level infrastructure service becomes an essential facility for the operators of first-level services, usually utility networks, on which the essential facility doctrine can be applied. There exist two major approaches to the 'efficient' pricing of essential input facilities: the 'efficient component' or parity pricing rule, and the Laffont-Tirole Ramsey pricing rule. The first rule is the principle that the holder of the bottleneck facility should offer its services at a price that yields it the same contribution that it would earn from performing the end-user or first-level service itself. The second rule recognizes the fact that the profit of the integrated incumbent is an increasing function of both the access charge and the final retail price. Both approaches accept the fact that these prices distort allocative efficiency. Just as is the case in most models in the theory of public finance, these rules are preoccupied with extracting money from the market so as to supply the government with sufficient funds. The public authority may appoint a regulator for this second type of models and instruct him to control the price-quality ratios on each level separately. In some countries, regulators are also appointed to restrain market failures. These regulators are gradually receiving tasks that aim to re-establish some structure in the de-integrated industry and to give some coherence, safety and equity in the service provision[3]. This public task of the regulator fits better in the design of governance under third-degree of government involvement. This third-degree government involvement is also applicable to the theory of public projects or club theory, see [8] and [17]. Our infrastructure service may be considered a specification of a public project. It also fits in the approach of Ellickson et al. [10] if the entire infrastructure service may be considered a competitive club. In that case, the authors show that a decentralised price-taking equilibrium exists with Pareto optimal allocations that belong to the core.

The solution concept proposed in this paper fits the third degree of government involvement and is consistent with the first degree. The public authority determines the rules of the game rather than the outcome of the game. These rules concern the coordination between the various levels by means of principal-agent contracts. Since the basic contract involves partners belonging to different domains, such a contract is called a PPP-

[3]The Office of the Rail Regulator, ORR, established in the U.K. in 1992, has the following functions: (i) the issue of licences to operate trains and networks; (ii) the enforcement of competition law; (iii) the approval of agreements for access by operators of railway assets to track and stations; (iv) customer protection and promotion of passengers' interests. The Rail Regulator is charged with the responsibility of carrying out these functions in a way which will: promote the use and development of a national railway network; minimise the regulatory burden; ensure commercial certainty and security; consider the environmental effect of railway services; consider the financial position of the Franchising Director and holders of licences.

contract. The guiding principle in such a contract is that the users of the lower-level service determine the higher-level service and pay for it according to their willingness to pay. The governance of an infrastructural industry is determined by the economic structure of the industry. Whenever there exists a complementarity between a public service and private services, a public private partnership may be established. The contract describes the way user-tariffs are defined and levied. Modern technical developments as computer chips make it possible for the industrial organization to discriminate between different groups of users giving different signals. If this is the case, we argue that:

(i) When the infrastructure service can be decomposed into a second-level public service and a complementary first-level, privately accountable service to its users, which service is possibly a marketable service, then the social benefit of the second-level service can be deduced from first-level service and a third-degree government involvement is possible.

(ii) When the infrastructure service cannot be decomposed into a second-level public service and a complementary first-level service to its users, then the social benefit is economically determined by Lindahl prices and has to be revealed by a political mechanism. A first-degree government involvement is required.

(iii) The proposed solution concept allows for a combination of both situations, generating and supporting economically efficient prices for the optimal level of an infrastructure service that fully finance the cost of this infrastructure.

(iv) The prices and the allocation related to this infrastructure service belong to a general equilibrium that is efficient and is shown to exist.

The idea of restricting preferences by some form of complementarity has been introduced by Mäler [16]. His concept of weak-complementarity differs strongly from our concept. Ebert [9] has followed Mäler's line of thought. The model introduced here is a modification of an earlier model introduced by Ruys [21], see also Ruys and van der Laan [22]. The earlier model deals with a so-called semi-public good, being a public good (e.g. infrastructure) that is characterized by the fact that its use is being complemented by certain private goods. For example, households and firms (transport sector) make use of the public good 'road' in combination with their private good 'car' or 'truck'. In Ruys and van der Laan [22] a model was developed in which the public good is financed by a lump-sum payment of the public sector and mark-ups on the market prices for the complementary private commodity good, collected by the private sector. The users are willing to pay these mark-ups because they are constrained on the use of the complementary private commodity by the limited availability of the public good. These mark-ups can be utilized for financing the costs of the public good. The problem is that the private sector must be willing to cooperate in collecting the mark-ups. This paper reflects the technical developments of the last decennium. By applying computer technology to monitoring and paying services, the willingness to pay mark-ups on the private commodity, can be utilized by an operational system forcing the users to pay directly for the use of the infrastructure.

The basic idea of this paper is that the use of infrastructure may be constrained by the size of the infrastructure, e.g. the consumption (in kilometers) of the private service 'car driving' (with price equal to the cost of car driving per kilometer) is constrained by the size of the road system. This constraint might be implicitly expressed in the consumer's utility function or the producer's production function, but in this paper we assume that the constraint is also given explicitly. This explicit formulation makes it possible to distinguish two effects: (i) the direct effect on utility because of the fact that the availability of the infrastructure service appears in the consumer's utility function, (ii) the indirect effect on utility through the weakening of the constraint. The direct effect has to be measured by the public sector by means of a political mechanism. It contributes to the lump-sum payment of the public sector to finance the infrastructure. The indirect effect will show up as what the user is willing to pay for the use of infrastructure if this use is constrained and can be measured by the road operator. If no user in the economy feels himself constrained in the use of the infrastructure, then the industry reduces to a pure public good industry with, if desired, Lindahl prices. On the other hand, when the direct effect is not relevant because users do not derive direct utility from the availability of infrastructure, then the industry reduces to a pure market industry and the infrastructure has to be financed only by the revenues from pricing the use of the infrastructure.

This paper is organized as follows. In Section 2 the economy is given in terms of the agents' characteristics. In Section 3 we state the first order conditions for Pareto efficient provision of infrastructure. The equilibrium structure to implement a Pareto efficient allocation is given in Section 4. An operational mode for inplementing a system for financing infrastructure is discussed in Section 5. Finally, technical details and existence of equilibium are discussed in the Appendix.

2 The economic ability structure

We consider a model of an economy with one type of public private service (pp-service), which is a (local) public service that is complementary to a specific private user service. The restriction to one pp-service is only for expositional reasons and is not essential for our approach. An example of a pp-service is an infrastructure service, such as a public road system that is utilized by owners of private cars and by firms transporting commodities. The complementarity of the pp-service may follow implicitly from utility functions or production functions in the economy, but since it is a central characteristic of the problem addressed, it is formulated explicitly. In this paper, the private service is interpreted as a private mobility or transport service that requires the complementary public road system. The size of the infrastructure effects the use of it by an individual agent. The use of this private service is measured on a one-dimensional scale, which will serve as the tax base for the use of the infrastructure. This measure may be refined arbitrarily to include various types and categories of public services and of user services. Examples of such an index are: the number of kilometers times the weight a specific vehicle uses the road system, or the number of liters of gasoline a specific vehicle needs to use the road system for some distance. A refinement of indices allows the user to substitute not only between types of vehicles, but also between modes of transport.

Besides the public private service, being a pair specifying the level of public infrastructure available in the economy and the private use of the infrastructure made by a specific user, we have a private commodity complementary to the use of the infrastructure, to be called the complementary private commodity, and n private commodities not related to the infrastructure, indexed by $j = 1, \ldots, n$ and to be called pure private commodities. There are $m + 1$ private agents, namely a set $H = \{2, \ldots, m\}$ of $m - 1$ consumers or households, indexed by $h = 2, \ldots, m$ and a set $F = \{0, 1\}$ of two private firms: one firm producing infrastructure indexed by $f = 0$ and one firm producing the private complementary commodity indexed by $f = 1$.

All households and the private commodity firm $f = 1$ are users of the public private service. We denote the set of users by I, i.e. $I = H \cup \{1\} = \{1, \ldots, m\}$. The pp-service of a user $i \in I$ is given by a pair (s^i, z) of nonnegative real numbers, where z denotes the level of infrastructure available in the economy and s^i denotes the private service or the use of the infrastructure, measured in terms of the chosen one-dimensional scale. For each user $i \in I$, there exist a nonnegative increasing function $q^i \colon \mathbb{R}_+ \to \mathbb{R}_+$, reflecting the individual, subjective constraint on the use s^i induced by the level z of the availability of the public infrastructure, i.e. for any pair (s^i, z) of the pp-service the inequality

$$s^i \leq q^i(z), \quad i \in I, \tag{1}$$

holds. For the execution of the use (s^i, z) of the pp-service, also the complementary private good is needed. For simplicity and without loss of generality we assume that one unit of the complementary private good is needed for every unit of the use s^i of the infrastructure. For example in case of car driving with gasoline as the complementary private good, the use of the infrastructure is measured in such a way that for each unit of the use of the infrastructure one unit of gasoline is needed. So, in the remaining of this paper, s^i denotes both the use of infrastructure of user i and the need of user i for the complementary private good.

Each user $h \in H$ has a utility function $u^h(x^h, s^h, z)$ on $X^h = \mathbb{R}_+^{n+2}$, where $x^h \in \mathbb{R}_+^n$ is the consumption of the n private goods, and as above, z is the level of infrastructure, available to all consumers, and s^h is the private use of the infrastructure by consumer h. Recall from above that this implies that the consumer also uses s^h units of the complementary private commodity. Since this consumption does not yield utility on its own, but only is needed to make use of the infrastructure, the consumption of this commodity does not appear explicitly in the utility function. Otherwise stated, the use s^h reflects both the use of the infrastructure and the consumption of the complementary private commodity: one may see it as a mobility service in which the private and public aspects melt together. In this formulation of the utility function, the infrastructure enters the utility function directly as a public availability service. One may discard this service from the utility function, reducing the utility function to a function $u^h(x^h, s^h)$ not depending on the level z of infrastructure. However, observe that in this case the level of infrastructure affects the utility indirectly through the constraint inequality (1). Consumer h is assumed to have an initial endowment $\omega^h \in \mathbb{R}_+^n$ of the n pure private commodities.

User 1 is the firm producing the private complementary commodity and is modelled by a transformation function $T^1\colon \mathbb{R}^{n+2} \to \mathbb{R}$ yielding the set of all feasible production plans $(x^1, -s^1, y^1)$ given by

$$T^1(x^1, -s^1, y^1) \leq 0, \tag{2}$$

where $x^1 \in \mathbb{R}_-^n$ is an n-vector of inputs of the pure private commodities, $s^1 \geq 0$ is the use made by the industry of the infrastructure, i.e. $-s^1$ is an input for the production sector, and $y^1 \geq 0$ is the output of the complementary private good. Observe that on the one hand the complementary good needed for the use of infrastructure is produced by the firm, while on the other hand, according to modern theories, see for instance Biehl [1], the use of infrastructure is incorporated as one of the inputs, and hence the firm needs also an amount of $-s^1$ units of the complementary good as input in the production process. So, the complementary private good is produced by the firm, but also appears as input: the pp-service of the infrastructure. Recall that the use of the infrastructure is constrained by the availability of the infrastructure by constraint (1) given by $s^1 \leq q^1(z)$.

Firm 0 produces the infrastructure and is modelled by a transformation function $T^0\colon \mathbb{R}^{n+1} \to \mathbb{R}$ yielding the set of all feasible production plans (x^0, z) given by

$$T^0(x^0, z) \leq 0, \tag{3}$$

where $x^0 \in \mathbb{R}_-^n$ is an n-vector of inputs of the n pure private commodities and $z \geq 0$ is the output of infrastructure.[4] With respect to the level of infrastructure, we assume that z indicates the yearly lease of the infrastructural capacity in real terms, including depreciation, maintenance, and so forth. Since the model only concerns one period and infrastructure is not built anew each period, the level z represents the level of infrastructure being available from the past, and the possible expansion or contraction of this infrastructure today and in the future. So, at an efficient production plan (x^0, z), the costs of the vector x^0 of inputs are the costs of the production of infrastructure that is accountable for today. If z is chosen to be zero, this means that one chooses for the fastest contraction of the infrastructure possible.

The economic ability structure is denoted by $\mathcal{E} = \{T^0, (T^1, q^1), (u^h, q^h, \omega^h),\ h \in H\}$. We assume that $\mathcal{E}$ is regular, i.e. the utility functions, transformation functions and the constraint functions are continuously differentiable, the utility functions are monotonically increasing and strictly quasi-concave, the transformation functions are strictly concave and satisfy $T^f(\underline{0}) = 0,\ f = 0, 1$, and all vectors ω^h are strictly positive.

3 Efficiency conditions in the ability structure

In this section efficiency conditions in the economic ability structure $\mathcal{E}$ are derived. An allocation e for the economy $\mathcal{E}$ is a collection of private consumption plans (x^h, s^h), $h \in H$, and production plans (x^0, z) and $(x^1, -s^1, y^1)$. For simplicity we restrict ourselves to interior allocations and so we restrict ourselves to allocations in which all quantities of consumption, inputs and outputs are not equal to zero and have the appropriate sign. In particular this convention implies that for any allocation e that $(x^h, s^h, z) \in X^h,\ h \in H$, holds by definition.

Definition 3.1 (Feasible allocation)
An allocation $e = \{(x^0, z),\ (x^1, -s^1, y^1),\ (x^h, s^h),\ h \in H\}$ is feasible for the economy $\mathcal{E}$ if

(i) *the inequalities (1) - (3) are satisfied,*

(ii) $\sum_{h\in H} x^h - x^0 - x^1 \leq \sum_{h\in H} \omega^h$,

(iii) $s^1 + \sum_{h\in H} s^h \leq y^1$.

Condition (i) includes the perceived constraints (1) on the use of infrastructure by the users (households and the complementarity commodity firm), and the production constraints (2) and (3). The other two conditions are the market clearing conditions. Condition (ii) states

[4] Only for simplicity the infrastructure firm is assumed not to be a user of infrastructure. However, the model can be easily generalized to the case that also this firm uses infrastructure.

that for the pure private commodities the total demand of the households and the firms is at most equal to the total initial endowments, and condition (iii) states that the total need for the complementary private good (measured in units of the use of infrastructure) is at most equal to the production of this commodity. In the following the set A denotes the set of all feasible allocations. For an allocation e, let $u^h(e) = u^h(x^h, s^h, z)$ denote the corresponding utility level of consumer h, $h \in H$.

Definition 3.2 (Efficient allocation)
An allocation e is efficient if it is feasible and there does not exist an other feasible allocation $e' \in A$, such that $u^h(e') > u^h(e)$ for all $h \in H$.

The efficiency conditions are derived from maximizing a social welfare function $W\colon \mathbb{R}^{m-1} \to \mathbb{R}$ assigning welfare level $W(u^h,\ h \in H)$ to utility levels u^h, $h \in H$, which is nondecreasing in u^h, $h \in H$, and strictly increasing in at least one u^h. From Definition 3.2 it follows that for any efficient allocation there exist nonnegative weights α_h, $h \in H$ with $\sum_{h \in H} \alpha_h = 1$ such that it maximizes the social welfare $\sum_{h \in H} \alpha_h u^h$ over the set of feasible allocations. Reversely, the necessary first order conditions for an efficient allocation follow from the maximization problem

$$\max\{\sum_{h=2}^{m} \alpha_h u^h(e) | e \in A\}. \tag{4}$$

Differentiating the Lagrangian function associated to this maximization problem with respect to the multipliers corresponding to the constraints given in (1), (2), (3) and the market clearing constraints (ii) and (iii) of Definition 3.1 we get the following complementarity restrictions between the constraints (on the left side) and the corresponding multipliers (on the right side), where $a \leq 0 \perp b \geq 0$ means $a \leq 0$, $b \geq 0$ and $a \cdot b = 0$,

$$s^i - q^i(z) \leq 0 \quad \perp \quad \beta_i \geq 0, \quad i \in I, \tag{5}$$

$$T^0(x^0, z) \leq 0 \quad \perp \quad \lambda_0 \geq 0, \tag{6}$$

$$T^1(x^1, -s^1, y^1) \leq 0 \quad \perp \quad \lambda_1 \geq 0, \tag{7}$$

$$\sum_{h \in H} x_j^h - x_j^0 - x_j^1 - \sum_{h \in H} \omega_j^h \leq 0 \quad \perp \quad \mu_j \geq 0, \quad j = 1, \ldots, n, \tag{8}$$

$$s^1 + \sum_{h \in H} s^h - y^1 \leq 0 \quad \perp \quad \mu_c \geq 0. \tag{9}$$

For simplicity in the remaining of the paper we assume that in an efficient allocation $e = \{(x^0, z),\ (x^1, -s^1, y^1),\ (x^h, s^h),\ h \in H\}$ both firms are active and that demand equals supply, i.e. the constraints on the left side of the complementarity conditions (6), (7), (8) and (9) hold with equality and the corresponding right side shadow prices are strictly positive. To clarify the further discussion, we rewrite the condition (5) explicitly as

$$\left.\begin{array}{lll} \beta_i = 0 & \text{if} & s^i - q^i(z) < 0, \\ \beta_i \geq 0 & \text{if} & s^i - q^i(z) = 0, \end{array}\right\} \quad i \in I, \tag{10}$$

showing that the shadow price β_i on the use of the infrastructure by user i is equal to zero when the constraint is not binding. Differentiating the Lagrangian associated with the maximization problem (4) with respect to the quantity variables (stated below between brackets) in $e = \{(x^0, z),\ (x^1, -s^1, y^1),\ (x^h, s^h),\ h \in H\}$ gives the first order conditions:

$$
\begin{array}{llll}
(x_j^h) & \alpha_h \frac{\partial u^h}{\partial x_j^h} - \mu_j & = 0,\ h \in H,\ j = 1, \ldots, n, & (11) \\
(s^h) & \alpha_h \frac{\partial u^h}{\partial s^h} - \beta_h - \mu_c & = 0,\ h \in H, & (12) \\
(x_j^0) & -\lambda_0 \frac{\partial T^0}{\partial x_j^0} + \mu_j & = 0,\ j = 1, \ldots, n, & (13) \\
(x_j^1) & -\lambda_1 \frac{\partial T^1}{\partial x_j^1} + \mu_j & = 0,\ j = 1, \ldots, n, & (14) \\
(s^1) & \lambda_1 \frac{\partial T^1}{\partial s^1} + \beta_1 + \mu_c & = 0, & (15) \\
(y^1) & -\lambda_1 \frac{\partial T^1}{\partial y^1} + \mu_c & = 0, & (16) \\
(z) & \sum_{h \in H} \alpha_h \frac{\partial u^h}{\partial z} + \sum_{h \in H} \beta_h \frac{\partial q^h}{\partial z} + \beta_1 \frac{\partial q^1}{\partial z} - \lambda_0 \frac{\partial T^0}{\partial z} & = 0. & (17)
\end{array}
$$

To focus on the first order condition for the production of infrastructure, we concentrate on equation (17). Choosing any private commodity j, the variables α_h can be solved from the equations (11) and the variable λ_0 from (13). Then substituting these expressions into equation (17) yields for any chosen $j = 1, \ldots, n$

$$\sum_{h \in H} \mu_j \frac{\partial u^h / \partial z}{\partial u^h / \partial x_j^h} + \sum_{h \in H} \beta_h \frac{\partial q^h}{\partial z} + \beta_1 \frac{\partial q^1}{\partial z} = \mu_j \frac{\partial T^0 / \partial z}{\partial T^0 / \partial x_j^0}. \tag{18}$$

Now we consider two cases. First, suppose that none of the user constraints $s^i \leq q^i(z)$ is binding, i.e. no user feels herself to be constrained in the use of infrastructure because of a (too) low level of infrastructure. Then according to the conditions in (10) all shadow prices β_i, $i \in I$, are equal to zero and for each $j = 1, \ldots, n$ equation (18) reduces to

$$\sum_{h \in H} \frac{\partial u^h / \partial z}{\partial u^h / \partial x_j^h} = \frac{\partial T^0 / \partial z}{\partial T^0 / \partial x_j^0}, \tag{19}$$

showing the well-known first order condition for a pure public good, i.e. for any given private commodity the sum over all consumers of the marginal rates of substitution between the public good and the private good is equal to the producer's marginal rate of transformation between the public good and the private good.

Second we consider the case that at least one of the users is constrained in the use of infrastructure. For any $j = 1 \ldots n$ and for any household $h \in H$ it follows from equations (11) and (12) that

$$\frac{\beta_h}{\mu_j} = \frac{\partial u^h / \partial s^h}{\partial u^h / \partial x_j^h} - \frac{\mu_c}{\mu_j}, \tag{20}$$

while from equations (14) and (16) it follows that

$$\frac{\mu_c}{\mu_j} = \frac{\partial T^1/\partial y^1}{\partial T^1/\partial x_j^1}, \tag{21}$$

and thus

$$\frac{\beta_h}{\mu_j} = \frac{\partial u^h/\partial s^h}{\partial u^h/\partial x_j^h} - \frac{\partial T^1/\partial y^1}{\partial T^1/\partial x_j^1}. \tag{22}$$

This shows that the ratio of consumer h's shadow price on the use of infrastructure and the shadow price of commodity j is equal to the difference of consumer h's marginal rate of substitution between the use of the infrastructure and the private good j and the marginal rate of transformation of the complementary private good produced by firm 1 and the input good j of firm 1. Recalling that an equal amount of units of the complementary good is needed for the use of the infrastructure, equation (22) reflects as a mark-up consumer h's willingness to pay for the use of infrastructure (in units of good j) above the marginal costs of the use to be paid for the complementarity good. Analogously it follows for the willingness to pay of the private commodity producer for the use of infrastructure as an input in its production process that

$$\frac{\beta_1}{\mu_j} = \frac{-\partial T^1/\partial s^1}{\partial T^1/\partial x_j^1} - \frac{\partial T^1/\partial y^1}{\partial T^1/\partial x_j^1}, \tag{23}$$

showing that the ratio of the producer's shadow price on the use of infrastructure and the shadow price of commodity j is equal to the difference of the producer's marginal rate of transformation between the use of the infrastructure and the private good j and the marginal rate of transformation of the produced complementary good and the private good j.[5] So, equation (23) reflects the producer's willingness to pay for the use of infrastructure above the marginal costs of the complementarity good needed for the use.

Substituting the equations (22) and (23) for β_h, $h \in H$, and β_1 in equation (18) we obtain

$$\sum_{h\in H} \frac{\partial u^h/\partial z}{\partial u^h/\partial x_j^h} + \sum_{h\in H} \left(\frac{\partial u^h/\partial s^h}{\partial u^h/\partial x_j^h} - \frac{\partial T^1/\partial y^1}{\partial T^1/\partial x_j^1} \right) \frac{\partial q^h}{\partial z} + \left(\frac{-\partial T^1/\partial s^1}{\partial T^1/\partial x_j^1} - \frac{\partial T^1/\partial y^1}{\partial T^1/\partial x_j^1} \right) \frac{\partial q^1}{\partial z}$$
$$= \frac{\partial T^0/\partial z}{\partial T^0/\partial x_j^0}. \tag{24}$$

So, when some of the constraints on the use are binding, the first order condition for the production of infrastructure says that with respect to any pure private good j it must hold that the sum of the marginal rates of substitution of all consumers plus the sum of

[5] Observe that $-\partial T^1/\partial s^1 > 0$, because $-s^1$ is an input and hence T^1 is increasing in $-s^1$.

the mark-ups the users are willing to pay is equal to the marginal rate of transformation of the producer of infrastructure, where the mark-up of a user equals her willingness to pay for the complementarity good beyond the cost of the complementarity good times the marginal relaxation of the constraint when more infrastructure is produced.

The main advantage of this economic organization is that it is possible to discriminate between users who are and who are not constrained by the infrastructure, because it can observe demand behavior for the complementary good. This information may solve the difficult problem of determining the individual contributions to the provision of a public good as infrastructure.

4 The institutional structure: a Public Private Partnership Equilibrium

In this section we formulate an institutional framework to implement an efficient allocation. This institution is the equilibrium framework in the private ownership economy. In this economy the two firms are profit maximizing firms and the consumers are utility maximizing agents. Moreover, we establish public private partnership which organizes and exploits the public private infrastructure service. This public private partnership is a principal-agent relation. The principal is the *public infrastructure agency*, the agent is the *infrastructure operator*. Both enter in a contractual relation in which the mutual intentions to be discussed below are specified. Notice that the agency has an institutional task and the operator a managerial task. The agency determines the optimal level of the infrastructure service, which is that level at which the sum of the individual prices for the availability of infrastructure and the sum of all mark-ups the users are willing to pay for the use of infrastructure, is equal to the marginal rate of transformation for infrastructure with respect to the numeraire commodity. The operator decides to buy the level of infrastructure from the infrastructure firm when he is able to collect enough contributions to cover the costs. These contributions come from two different sources. First, the agency collects the valuation of the users of the availability of infrastructure as a pure public good and pays the total amount of these valuations to the operator. Second, the operator is empowered by the agency to set tariffs, regulated by the agency, on the use of the infrastructure. The profit of the operator is the difference between the revenues from these two sources and the costs of providing the determined level of infrastructure. It will be shown that under some conditions on the constraint functions in equilibrium this profit is nonnegative, so that the operator is willing to perform his task.

As usual in a private ownership economy, all profits are distributed amongst the consumers. So, let ϕ^{hf} be the share of consumer h, $h \in H$, in the profit of firm f, $f = 0, 1$

and ϕ^h the share of h in the profit of the operator. All shares are assumed to be nonnegative and satisfy $\sum_{h\in H}\phi^{hf}=1$ for $f=0,1$, and $\sum_{h\in H}\phi^h=1$.

To define the equilibrium concept, let $p\in\mathbb{R}^n$ be the vector of prices of the n pure private commodities, p_y the price of the complementary private commodity and p_z the price of one unit of the infrastructure. Commodity one is assumed to serve as the numeraire commodity with price $p_1=1$. According to the well-known concept of Lindahl equilibrium (see [18] or [3]), we also define for each $h\in H$ a personal price p^h as the public good contribution consumer h has to pay for each unit of available infrastructure. Moreover for each user $i\in I$ we define a tariff t^i to be paid for each unit of use of the infrastructure. This tariff reflects the shadow price of the quantity constraint $q^i(z)$ on the use of infrastructure. This reasoning has analogies in fixed price theory, from which it is well-known that quantity-constrained allocations can be sustained by virtual taxation, i.e. quantity constraints in an equilibrium under fixed prices can be replaced by virtual taxes and a redistribution of the revenues of the taxes (see e.g. Neary and Roberts [19] and Cornielje and van der Laan [5], see also Ruys [20]). Finally, let m^h denote the income of consumer h, $h\in H$, π^f the profit of firm f, $f=0,1$ and π the operator's profit, all to be defined later. In the following m^H denotes the collection of incomes m^h, $h\in H$, p^H the collection of personal contributions p^h, $h\in H$ and t^I the collection of tariffs t^i, $i\in I$.

We first consider the problem of the public agency. This agency has to determine on the personal contributions p^H and user tariffs t^I and the production price p_z. Given some feasible allocation $e=\{(x^0,z),\ (x^1,-s^1,y^1),\ (x^h,s^h),\ h\in H\}$, for each $h\in H$ the agency sets the individual price p^h to be paid for each unit of the level z of infrastructure in this allocation equal to

$$p^h=\frac{\partial u^h/\partial z}{\partial u^h/\partial x_1^h}, \tag{25}$$

being the marginal rate of substitution of consumer h between the level z of the infrastructure and her consumption x_1^h of private good 1. With respect to the tariffs, first observe that according to Definition 3.1 any feasible allocation satisfies the quantity constraints on the use of infrastructure, i.e. $s^i\leq q^i(z)$ for all $i\in I$. According to the reasoning given above, for each user $i\in I$ the tariff t^i to be paid for each unit of use of the infrastructure is set equal to the shadow price of the quantity constraint $q^i(z)$. So, the tariff is set equal to the willingness to pay for the use of infrastructure above the price p_y of the complementary commodity and thus, for $h\in H$, t^h is set equal to

$$t^h=\begin{cases}=0 & \text{if}\quad s^h<q^h(z),\\ =\frac{\partial u^h/\partial s^h}{\partial u^h/\partial x_1^h}-p_y & \text{if}\quad s^h=q^h(z).\end{cases} \tag{26}$$

and for firm 1 the tariff t^1 is set equal to

$$t^1 = \begin{cases} = 0 & \text{if} \quad s^1 < q^1(z), \\ = \frac{-\partial T^1/\partial s^1}{\partial T^1/\partial x_1^1} - p_y & \text{if} \quad s^1 = q^1(z). \end{cases} \tag{27}$$

Finally, the production price is set equal to

$$p_z = \sum_{h \in H} p^h + \sum_{i \in I} t^i \frac{\partial q^i}{\partial z}. \tag{28}$$

.

Given the feasible allocation $e = \{(x^0, z),\ (x^1, -s^1, y^1),\ (x^h, s^h),\ h \in H\}$, and the prices p^H and tariffs t^I set by the agency, the operator exploits the public private infrastructure service by buying from the infrastructure firm 0 the level z of infrastructure against price p_z per unit of infrastructure. To fund the costs of the infrastructure the operator collects revenues from two sources. The first source consist of the consumers' contributions p^h, $h \in H$, per unit of the level of infrastructure and the second one are the revenues from the tariffs t^i, $i \in I$, the users have to pay for the use of the infrastructure. The operator's profit is the difference between the revenues of exploiting the infrastructure and the cost of providing the infrastructure and is therefore given by

$$\pi(z, p^H, t^I) = \sum_{h \in H} p^h z + \sum_{i \in I} t^i s^i - p_z z. \tag{29}$$

Observe that $p^h = 0$ when the utility of consumer h does not depend on z, i.e. when $u^h = u^h(x^h, s^h)$. When this holds for all consumers equation (29) reduces to

$$\pi(z, p^H, t^I) = \sum_{i \in I} t^i s^i - p_z z. \tag{30}$$

It will shown below that, under some conditions on the constraint functions, in equilibrium the operator's profit is nonnegative.

The firms are profit maximizing. Given prices $p \in \mathbb{R}^n$ and $p_z \in \mathbb{R}$ the maximization problem for firm 0 becomes

$$\max_{x^0, z} \sum_{j=1}^{n} p_j x_j^0 + p_z z \quad \text{s.t.} \quad T^0(x^0, z) \leq 0. \tag{31}$$

The solution to this problem is denoted by

$$\Big(x^0(p, p_z),\ z(p, p_z)\Big) \in \mathbb{R}^n_- \times \mathbb{R}_+,$$

specifying the demands for the private commodities and the supply of infrastructure. The corresponding profit is given by

$$\pi^0(p, p_z) = \sum_{j=1}^{n} p_j x_j^0(p, p_z) + p_z z(p, p_z).$$

Firm 1 is a user of the infrastructure and thus has to pay the price p_y for each unit of the complementary private good and an additional tax t^1 for each unit of use. Furthermore, this firm produces the complementary commodity. So, given prices $p \in \mathbb{R}^n_+$, $p_y \in \mathbb{R}$ and tariff $t^1 \geq 0$ the maximization problem for firm 1 becomes

$$\max_{x^1, -s^1, y^1} \sum_{j=1}^{n} p_j x^1_j - p_y s^1 - t^1 s^1 + p_y y^1 \text{ s.t. } T^1(x^1, -s^1, y^1) = 0. \tag{32}$$

Observe that the tariff to be paid on the use s^1 replaces the quantity constraint on the use. The solution to this problem is denoted by

$$\Big(x^1(p, p_y, t^1),\ -s^1(p, p_y, t^1),\ y^1(p, p_y, t^1)\Big) \in \mathbb{R}^n_- \times \mathbb{R}_- \times \mathbb{R}_+,$$

specifying the demand and supplies of the pure private commodities, the demand for the complementary commodity related to the use of infrastructure and the supply of the complementary commodity produced by the firm. The corresponding profit is given by

$$\pi^1(p, p_y, t^1) = \sum_{j=1}^{n} p_j x^1_j(p, p_y, t^1) - (p_y + t^1) s^1(p, p_y, t^1) + p_y y^1(p, p_y, t^1).$$

Finally we consider the consumers. The expenditures of consumer h consists of the costs of her consumption of the pure private commodities, the contribution p^h she has to pay for each unit of the level of infrastructure to the public agency, and his expenditures for the use of infrastructure, being the price p_y to be paid for each unit of the complementary private good and the additional tax t^h to be paid for each unit of use to the operator. So, given prices $p \in \mathbb{R}^n$, $p_y \in \mathbb{R}$ and $p^h \geq 0$, tariff $t^h \geq 0$ and income $m^h > 0$, consumer h solves the utility maximizing problem

$$\max_{x^h, s^h, z} u^h(x^h, s^h, z) \text{ s.t. } \sum_{j=1}^{n} p_j x^h_j + p_y s^h + t^h s^h + p^h z \leq m^h. \tag{33}$$

Again the tariff to be paid on the use s^h replaces the quantity constraint on the use. The solution to this problem is denoted by

$$\Big(x^h(p, p_y, t^h, p^h, m^h),\ s^h(p, p_y, t^h, p^h, m^h),\ z^h(p, p_y, t^h, p^h, m^h)\Big) \in \mathbb{R}^n_+ \times \mathbb{R}_+ \times \mathbb{R}_+,$$

specifying the demands for the private commodities, the demand for the complementary commodity, being equal to the use of infrastructure, and the 'demand' of infrastructure of consumer h, being the level of infrastructure that maximizes her utility given the price p^h to be paid for each unit of z.

We are now able to define a Public Private Partnership Equilibrium (PPPE) for the private ownership economy $\mathcal{E}^P = \{T^0,\ (T^1, q^1),\ (u^h, q^h, \omega^h, \phi^{h0}, \phi^{h1}, \phi^h),\ h \in H\}$ with Public Private Partnership.

Definition 4.1 Public Private Partnership Equilibrium (PPPE)
A **Public Private Partnership Equilibrium** *for the private ownership economy $\mathcal{E}^P$ with Public Private Partnership is a feasable allocation $e = \{(x^0, z),\ (x^1, -s^1, y^1),\ (x^h, s^h),\ h \in H\}$, a collection m^H of incomes, commodity prices $(p, p_y, p_z) \in \mathbb{R}^{n+2}$, personal infrastructure prices p^H, tariffs t^I and profits π^0, π^1 and π such that*

(i) p^H *satisfies (25),*

(ii) t^I *satisfies (26), respectively (27),*

(iii) $\pi^0 = \pi^0(p, p_z)$, $\pi^1 = \pi^1(p, p_y, t^1)$ *and* $\pi = \pi(z, p^H, t^I)$,

(iv) *for all* $h \in H$, $m^h = \sum_{j=1}^n p_j \omega_j^h + \phi^{h0}\pi^0 + \phi^{h1}\pi^1 + \phi^h \pi$,

(v) *for all* $h \in H$, $x^h = x^h(p, p_y, t^h, p^h, m^h)$ *and* $s^h = s^h(p, p_y, t^h, p^h, m^h)$,

(vi) *for all* $h \in H$, $z^h(p, p_y, t^h, p^h, m^h) = z$,

(vii) $x^0 = x^0(p, p_z)$ *and* $z = z(p, p_z)$

(viii) $x^1 = x^1(p, p_y, t^1)$, $s^1 = s^1(p, p_y, t^1)$ *and* $y^1 = y^1(p, p_y, t^1)$,

(ix) $\sum_{h\in H} x^h - x^0 - x^1 = \sum_{h \in H} \omega^h$,

(x) $\sum_{i \in I} s^i = y^1$,

(xi) p_z *satisfies (28).*

First, observe that an equilibrium allocation is defined to be a feasible allocation and thus all constraints of Definition 3.1 are satisfied, in particular the users' demands for the use of infrastructure satisfy their quantity constraints. Next, the first two conditions (i) and (ii) say that the correct personal prices and tariffs are determined, i.e. the personal prices and tariffs satisfy the first order conditions for efficiency. Conditions (iii) and (iv) say that the profits and incomes are correctly specified. Conditions (v) and (vi) say that in the equilibrium allocation the consumptions of the consumers for the pure private commodities and the use of infrastructure are their utility maximizing consumptions and that for each consumer h the level of infrastructure is equal to the optimal level of infrastructure maximizing the utility of consumer h given the personal price to be paid. This corresponds to the well-known Lindahl equilibrium condition for a public good in a pure public good economy. Conditions (vii) and (viii) say that in the equilibrium allocation the production plans of the two firms are profit maximizing. Conditions (ix) and (x) are the market clearing conditions for the private commodities and the complementary commodity respectively. Finally condition (xi) says that the sum of the personal prices plus the sum of the mark-ups the users are willing to pay for the use of infrastructure equals the price per unit of infrastructure, so that the first order efficiency equation (24) holds. Observe that in case none of the quantity constraints on the use is binding, this reduces to the standard condition in a pure public good Lindahl equilibrium saying that the sum of the personal prices is equal to the price of infrastructure. Observe that the profits appear in the incomes of the consumers and depends on the consumers' decision. Therefore the incomes and profits are taken explicitly in the definition of the equilibrium. Furthermore, it should be observed that the level of infrastructure is determined by the profit maximizing infrastructure firm yielding $z = z(p, p_z)$.

Once more we would like to stress the fact that the public agency only determines the individual prices p^H and the tariffs t^I. Doing this correctly, in equilibrium all consumers choose simultaneously the correct level of infrastructure, being the level chosen by firm 0 as his profit maximizing output. We also want to stress again that in equilibrium the rationing constraints on the use of infrastructure are satisfied, because of condition (ii), saying that in equilibrium the tariffs are set equal to the shadow price of the use of infrastructure when facing the constraint, i.e. the unconstrained demand of a user just equals the constraint when the tariff is positive and is at most equal the constraint when the tariff is zero.

From the conditions in Definition 4.1 and the utility and profit maximizing behavior of the private agents it follows immediately that a PPPE allocation satisfies all first order conditions for an efficient allocation as derived in the previous section. So, taking the second order conditions for granted, the following corollary follows straightforwardly.

Corollary 4.2
An PPPE allocation is efficient.

Finally we consider the operator's profit. A necessary condition for the implementation of a PPPE by a Private Public Partnership relation between the public agency and the operator ownership is that the operator's profit is nonnegative. Clearly, otherwise no operator willing to sign a contract for exploiting the infrastructure can be found. Therefore we consider again the operator's profit given by equation (29). In equilibrium we have that the prices satisfy equilibrium condition (xi) and hence $\sum_{h \in H} p^h - p_z = -\sum_{i \in I} t^i \frac{\partial q^i}{\partial z}$. Substituting this in (29) and using the equilibrium properties (26) and (27) saying that $s^i = q^i(z)$ if $t^i > 0$, it follows that

$$\pi = \sum_{i \in I} t^i s^i (1 - \epsilon^i(z)), \tag{34}$$

where $\epsilon^i(z) = \frac{\partial q^i}{\partial z} \cdot \frac{z}{q^i(z)}$ is user i's individual *infrastructure elasticity of the demand* for the complementary private good at the infrastructure level z. So, in equilibrium the operator's profit follows from the tariffs and the elasticities of the demands for the use of infrastructure and does not depend on the individual prices p^h and the price p_z, meaning that the 'pure public good' feature of the infrastructure does not affect the operator's profit. From (34) it follows immediately that the operator's profit is equal to zero if for all i, $\epsilon^i(z) = 1$ or $t^i = 0$. In particular this holds when all constraint functions are linear functions given by $q^i(z) = a^i z$ for some $a^i > 0$, $i \in I$ and hence all elasticities are equal to one. When q^i is a strict concave function with $q^i(0) \geq 0$, then $\epsilon^i(z) > 1$ and user i provides a nonnegative contribution to the operator's profit. So, a sufficient condition for the public private partnership structure is that the constraint functions are weakly concave nonnegative functions, guaranteeing that the operator is making nonnegative profits, i.e. the participation constraint of the operator is satisfied and hence he is willing to participate in the relationship.

To conclude this section it should be noticed that for the implementation of a PPPE still the informational problem of finding the individual prices and the tariffs has to be solved. To do so, some incentive mechanism is needed for the users to reveal this information. Instead of doing so, in the next section we discuss an operational implementation of a second-best equilibria based on the features of the model expressed by the first order conditions, namely that the consumers and private producer are willing to pay for the use of infrastructure. The existence proof of a PPPE is given in the Appendix.

5 Inefficiency costs of operational structures

In this section we discuss a practical possibility to implement a system for financing infrastructure based on the willingnesses to pay because of the perceived constraints on the use of infrastructure. In order to focus on this issue, we make some simplifying assumptions. Firstly, we assume that the public sector has solved the informational problem with respect to the individual prices p^h reflecting the marginal rate of substitution $\frac{\partial u^h/\partial z}{\partial u^h/\partial x_1^h}$, $h \in H$. Staying away from this problem, alternatively we may assume that the utilities only depend on the use of infrastructure, but not on the level of availability, i.e. $u^h = u^h(x^h, s^h)$ for all $h \in H$. Doing so, we focus on the impact of the constraint on the use of infrastructure. Furthermore, with respect to these constraints we assume for all $i \in I$ that $q^i(z) = a^i z$ for some $a^i > 0$. So, all elasticities are equal to one and in equilibrium the operator's profit is equal to zero.

Assumption 5.1
For the private ownership economy with Public Private Partnership $\mathcal{E}^P$ the following holds:

(i) *(no direct utility effects) for all $h \in H$ it holds that $u^h = u^h(x^h, s^h)$,*

(ii) *(unit constraint elasticities) for all $i \in I$ it holds that $q^i(z) = a^i z$, for some $a^i > 0$.*

Under Assumption 5.1 it follows that all individual prices p^h are equal to zero, while the derivatives of the constraint functions q^i to z are equal to the coefficients a^i, $i \in I$. So condition (xi) of the PPPE definition 4.1 reduces to

$$\sum_{i \in I} a^i t^i = p_z, \tag{35}$$

with t^h, $h \in H$, and t^1 satisfying (26), respectively (27). So, the price p_z is given as soon as the tariffs t^i are known. Of course, here we still encounter the informational problem that the willingnesses to pay that determine the tariffs t^i are not known by the public agency. The agency may, however, be advised by the operator, who can deduce prices from actual behavior of the users. Here we may think that the agency is able to classify the users into a number of more or less homogeneous groups, so that for each group the tariff to be paid for the use of infrastructure can be determined by considering the representative user. In the remaining of this section we will consider the extreme case of such an implementation, namely the case in which all users are treated in the same way. This implementation of a payment system is called *infrastructure pricing*.[6] Of course such an implementation is a second best solution. In this section we discuss on the loss of efficiency under such an implementation.

Under the regime of infrastructure pricing each user has to pay a uniform tariff for the use of infrastructure as far as the use is above some base level $\widehat{s}$.[7] Nowadays such a system can easily be implemented by using electronic systems of payments. In the following, let $s^{i+} = \max[0,\ s^i - \widehat{s}]$, $i \in I$, so s^{i+} is the use as far as it is above the base level. To have an effective system, the base level is chosen is such a way that the optimal choice s^i of user i will be above the base level for at least a substantial fraction of the users. Moreover we assume that $\widehat{s} < q^i(z)$ for all $i \in I$. The latter assumption is innocent because in practice this will be true for almost every user. Let t be the tariff to be paid per unit of use above the base level. For the private producer the profit maximization problem becomes

$$\max_{x^1, -s^1, y^1} \sum_{j=1}^{n} p_j x_j^1 - p_y s^1 - t s^{1+} + p_y y^1 \text{ s.t. } T^1(x^1, -s^1, y^1) \leq 0 \text{ and } s^1 \leq q^1(z). \tag{36}$$

Observe that in this situation of a uniform tariff system the tariff to be paid for the use does not guarantee that the quantity constraint becomes redundant. Analogously, the utility maximization problem of consumer h, $h \in H$, becomes

$$\max_{x^h, s^h} u^h(x^h, s^h) \text{ s.t. } \sum_{j=1}^{n} p_j x_j^h + p_y s^h + t s^{h+} \leq m^h \text{ and } s^h \leq q^h(z). \tag{37}$$

[6]The terminology reflect the current debate in the Netherlands about introducing a system of road pricing.

[7]The Dutch government considers to implement a road pricing system in which the users only have to pay during rush hours. The free use outside rush hours can be seen as the use up to the base level.

Considering this operational payment system, we want to discuss on the following questions:

(i) What can be said about the revenues for the public private partnership?

(ii) What can be said about the efficiency of this system?

(iii) What are the consequences of this pricing system for the use of infrastructure?

Concerning the first question, the revenues of the operator depend on the uniform tariff t. So, let $R(t)$ denote the revenues at tariff t, then we have that

$$R(t) = \sum_{i \in I} t s^{i+}. \tag{38}$$

Clearly, while $R(t) = 0$ for $t = 0$ the revenues may be expected to increase and to reach a maximum at certain value $\widehat{t}$, and to decrease when t will be further increased, because typically at the solutions of the maximization programs (36) and (37) the optimal values of s^{i+} will go to zero for t large enough. Now, let z^* be the efficient level of infrastructure in the PPPE allocation and let p_z^* be the corresponding equilibrium price of infrastructure. Now it is reasonable to assume that the maximum revenue satisfies

$$R(\widehat{t}) > p_z^* z^*. \tag{39}$$

Then, there exists a tariff $t^* < \widehat{t}$ such that the revenues are equal to the costs of implementing the optimal level of infrastructure.

This brings us to the second question. Under condition (39) the tariff $t^* > 0$ is such that the revenues are just equal to the costs $p_z^* z^*$ of the efficient level z^*. So, in general the system of a uniform tariff is able to sustain the efficient level of producing new infrastructure. Then the informational problems of the agency of finding the optimal individual tariffs are reduced to the more simple problem of finding the correct uniform tariff t^*. Using market surveys this does not seem to be too difficult. Of course, the pricing rule does not sustain an efficient allocation, because it does not discriminate between users. More precisely, the uniform pricing rule does not take into account the individual mark-ups expressing the willingnesses to pay and so the efficiency conditions (26) and (27) are not satisfied. Therefore the uniform pricing rule seems to be quite reasonable as an approximate solution to the socially optimal individual tariffs.

To answer the third question, we first consider the efficient equilibrium as given in Definition 4.1 and partition the set of users into three groups. To do so, for $i \in I$, let s^{i*} be the values of the use of infrastructure in the PPPE and recall that the use is free of charge up to the base level $\widehat{s}$. We now partition the set of users into three subsets by defining

$$\begin{aligned} I^1 &= \{i \in I \mid s^{i*} \leq \widehat{s}\}, \\ I^2 &= \{i \in I \mid \widehat{s} < s^{i*} < q^i(z^*)\}, \\ I^3 &= \{i \in I \mid \widehat{s} < s^{i*} = q^i(z^*)\}. \end{aligned} \tag{40}$$

Since we assumed that $\widehat{s} < q^i(z^*)$ for all $i \in I$, in the efficient PPPE we have that the individual tariff of user i is equal to zero when $i \in I^1 \cup I^2$, while the tariffs are positive for the users in I^3. Now, suppose that the optimal uniform tariff t^* will be used instead of the individual tariffs. Under this tariff also the social efficient level z^* is produced. Observe that in both situations the operator's profit is zero and hence there are no direct effects on the incomes of the consumers. Ignoring the effects of replacing individual tariffs by a uniform pricing rule on the prices of the other private commodities, the maximization problems (36) and (37) only differ from the optimal programs (33) and (32) with respect to the pricing on the use of infrastructure and the qunatity constraints, which in the PPPE allocation are redundant in the latter. We can now consider the effects for the users in the three groups defined above.

Clearly, for the group of users in I^1 there is no difference between the uniform tariff system and the optimal system of individual mark-ups. Because the use of these users is below the base level, in both systems they do not have to pay for the use of the infrastructure. So, as a result, also the use of infrastructure does not change for the users in this group. Also for most of the users in group I^3 there is no essential difference. At the efficient PPPE, all agents in this group make use of the infrastructure up to their quantity constraints and hence have to pay their individual tariff. Of course, in the PPPE some users have to a high tariff, and others a low tariff, reflecting their individual preferences. So, under a uniform system the users with a high willingness to pay are better off, the users with a low willingness to pay are worse off. Some of the latter users may reconsider their use and reduce their use below their constraint level. So, maybe some of the users become unconstrained and hence move from group I^3 to I^2. However, for all other users in I^3 the use will remain equal to their constraint and hence also for these users the use of infrastructure does not change.

Finally, we consider the users which are in group I^2 under the system of individual tariffs. These users are not constrained and do not have to pay for the use under the PPPE set-up. However, under the uniform pricing rule they have to pay for the use above the base level. As a consequence it may be expected that they will reduce their use. However, the use is at most reduced to their use at the base level, because at that level they will switch from group I^2 into I^1 and the use becomes free.

Summarizing we have the following. For the users in group I^1 there is no difference in what they have to pay and their use. The users in group I^2 have free use in the PPPE set-up and have to pay under the uniform pricing rule, resulting in some reduction of the use, but not further than their use at the base level. The users in group I^3 with high willingness to pay have to pay less in the uniform system and will be better off. They will not change their use: in both situations they will use up to the constraint level. The users in group I^3 with low willingness to pay are worse off under the uniform system. The users with very low willingness to pay may not be willing to use anymore up to their constraint level and will reduce their use below that level. In most situations this will be the case for a very small group of users in I^3. When this is the case we may conclude from this qualitatively analysis that the distortionary effects from using the uniform tariff system instead of the individual tariffs are very small. Because it solves a lot of the informational problems, the uniform system seems to be a very good alternative for the public private partnership as a system for financing the production of new infrastructure. It is easy to implement, the level of the tariff can be chosen such that the revenues are just enough to cover the costs of the socially efficient level of production and the distortionary effects are small.

6 Appendix: Existence of equilibrium

To prove the existence of a PPPE, first observe that all profits and incomes are homogeneous of degree one in the prices and tariffs (p, p_y, p_z, p^H, t^I) and that all consumption and production decisions of the agents are homogeneous of degree zero in (p, p_y, p_z, p^H, t^I). Denoting $\zeta = (p, p_y, p_z, p^H, t^I)$, we therefore restrict the collection of the $(n + 2m + 1)$-dimensional vectors ζ of prices and tariffs to the $(n + 2m)$-dimensional unit simplex $S^{n+2m} = \{(\zeta \in \mathbb{R}^{n+2m}_{+} \mid \sum_{k=1}^{n+2m+1} \zeta_k = 1\}$, where $p_k = \zeta_k = p_k$ for $k = 1, \ldots, n$, $p_y = \zeta_{n+1} = p_y$, $p_z = \zeta_{n+2} = p_z$, $p^h = \zeta_{n+1+h} = p^h$ for $h = 2, \ldots, m$ and $t^i = \zeta_{n+1+m+i} = t^i$ for $i = 1, \ldots, m$. Further, let $A \in \mathbb{R}^n_{+}$, $B > 0$ and $C > 0$ be such that $A_j > \sum_{h \in H} \omega^h_j$ for all j, $B > \max\{z \mid T^0(x^0, z) \leq 0 \text{ and } -x^0 \leq A\}$ and $C > \max\{y^1 \mid T^1(x^1, -s^1, y^1) \leq 0,\ -x^1 \leq A \text{ and } s^1 \leq q^1(B)\}$. So, A is greater than the total initial endowment and B and C exceed the maximal possible production of the public good and private good, respectively. Furthermore, let $K^0 = \{(x^0, z) \mid T^0(x^0, z) = 0 \text{ and } -x^0 \leq A\}$ and $K^1 = \{(x^1, -s^1, y^1) \mid T^1(x^1, -s^1, y^1) = 0,\ -x^1 \leq A \text{ and } s^1 \leq q^1(B)\}$. We now make the following assumptions.

Assumption 6.1
The private ownership economy $\mathcal{E} = \{T^0,\ (T^1, q^1),\ (u^h, q^h, \omega^h, \phi^{h0}, \phi^{h1}, \phi^h),\ h \in H\}$ is regular, i.e. the utility functions, transformation functions and the constraint functions are continuously differentiable, the utility functions are monotonically increasing and strictly quasi-concave, the transformation functions are strictly concave and satisfy $T^f(\underline{0}) = 0$, $f = 0, 1$, for all $h \in H$, ω^h_1 is strictly positive and $\omega_j = \sum_{h \in H} \omega^h_j > 0$ for all $j = 1, \ldots, n$.

Instead of assuming that $\omega^h_j > 0$ for all h and all j the weaker condition as stated in Assumption 6.1 is sufficient.

Assumption 6.2
The private ownership economy $\mathcal{E} = \{T^0,\ (T^1, q^1),\ (u^h, q^h, \omega^h, \phi^{h0}, \phi^{h1}, \phi^h),\ h \in H\}$ *the following holds:*

(i) *For any* $(x^0, z) \in K^0$ *it holds that* $z = 0$ *when* $x_1^0 = 0$*,*

(ii) *For any* $(x^1, -s^1, y^1) \in K^1$ *it holds that* $y^1 = 0$ *when* $x_1^1 = 0$*,*

(iii) T^1 *satisfies that* $\frac{\partial T^1/\partial y^1}{-\partial T^1/\partial s^1} < 1$ *at any* $(x^1, -s^1, y^1)$ *such that* $s^1 \geq y^1$*.*

(iv) T^0 *satisfies that for any* $\varepsilon > 0$ *there exists* $\lambda > 0$ *such that at any* $(x^0, z) \in K^0$ *with* $z < \lambda$ *it holds that* $\frac{\partial T^0/\partial z}{\partial T^0/\partial x_1^0} > \varepsilon$*.*

The first two assertions say that no output can be produced without any input of commodity 1. The third assertion says that the demand for use by firm 1 will never exceed the supply of the complementary good by firm 1, guaranteeing that the net supply of the complementary commodity is nonnegative. The last assertion says that under profit maximization the production is strictly positive at any ζ satisfying $\frac{p_z}{p_1} > 0$.

Assumption 6.3
For all $i \in I$*, the function* q^i *is concave and* $\frac{\partial q^i(z)}{\partial z}$ *continuous in* z *and bounded at* $z = 0$*.*

The concavity of the constraint functions guarantees that in equilibrium the operator's profit is is nonnegative. The boundedness condition of the derivatives at $z = 0$ is a technical condition implying that the mark-ups the users are willing to pay are bounded.

We now construct a function from the $(n+2m)$-dimensional unit simplex to $\mathbb{R}^{n+2m+1}$ and show that this function has a stationary point. It then remains to show that such a stationary point yields an equilibrium.

Let $\zeta = (p, p_y, p_z, p^H, t^I) \in S^{n+2m}$ be a vector of prices and tariffs with $p_1 > 0$. Then under Assumption 6.1 and Assumption 6.2, part (i) the profit maximizing problem

$$\max px^0 + p_z z \text{ s.t. } (x^0, z) \in K^0$$

has a uniqe solution, to be denoted by $(x^0(\zeta), z(\zeta))$, with corresponding profit $\pi^0(\zeta)$. We make the following additional assumption.

Assumption 6.4
Let $\zeta^k \in S^{n+2m}$*,* $k = 1, \ldots,$ *be a sequence converging to some* ζ *with* $p_1 > 0$ *and* $p_z = 0$*. Then* $\lim_{k\to\infty} \frac{z(\zeta^k)}{p_z} = N(\zeta)$ *for some real number* $N(\zeta) > 0$*.*

The assumption implies that at any positive price of the first private commodity the supply of infrastructure goes to zero when the output price p_z goes to zero. Clearly, this holds under Assumption 6.1, part (i). However, for technical reasons we assume a little bit more, namely that the order of convergence to zero of $z(\zeta)$ when p_z goes to zero is equal to one.

Under Assumption 6.1 and 6.2, part (ii), also the profit maximization problem

$$\max px^1 - p_y s^1 - t^1 s^1 + p_y y^1 \text{ s.t. } (x^1, -s^1, y^1) \in K^1$$

has a unique solution for $p_1 > 0$, to be denoted by $(x^1(\zeta), s^1(\zeta), y^1(\zeta))$, with corresponding profit $\pi^1(\zeta)$. By the regularity assumption we have that all solutions and the profits are continous in ζ and that also both $\pi^0(\zeta)$ and $\pi^1(\zeta)$ are nonnegative for all $\zeta \in S^{n+2m}$. Furthermore, we define the operator's profit at ζ by

$$\pi(\zeta) = \sum_{i \in I} t^i q^i(z(\zeta))(1 - \epsilon^i(z(\zeta)),$$

where $\epsilon^i(z(\zeta))$ is user i's infrastructure elasticity of demand at level $z(\zeta)$. Clearly, π is continuous in ζ and by Assumption 6.3 also $\pi(\zeta)$ is nonnegative for all $\zeta \in S^{n+2m}$.

For $h \in H$, the income $m^h(\zeta)$ of consumer h given by

$$m^h(\zeta) = p^\top \omega^h + \phi^{h0}\pi^0(\zeta) + \phi^{h1}\pi^1(\zeta) + \phi^h \pi(\zeta)$$

is continuous in ζ and nonnegative for all $\zeta \in S^{n+2m}$. We now consider the restricted utility maximizing problem

$$\max u^h(x^h, s^h, z) \text{ s.t. } \begin{cases} px^h + p_y s^h + t^h s^h + p^h z \le m^h(\zeta), \\ x^h \le A,\ s^h \le C,\ z^h \le B. \end{cases}$$

Under the regularity assumption this problem has a uniqe solution, to be denoted by $x^h(\zeta)$, $s^h(\zeta)$ and $z^h(\zeta)$, where $z^h(\zeta)$ is consumer h's optimal level of infrastructure at ζ. Under the regularity condition we have from the fact that $\omega_1^h > 0$ that $m^h(\zeta)$ is positive at any ζ with $p_1 > 0$, while because of the monotonicity of the utility function and the constraints $x^h(\zeta) \le A$, $s^h(\zeta) \le C$ and $z^h(\zeta) \le B$ it follows that there exists some (small) $\delta > 0$ such that $x_1^h(\zeta) = A_1$ when $p_1 < \delta$, which implies that in any PPPE we must have that $p_1 \ge \delta$. Therefore we restrict the set of vectors ζ to the set $S_\delta^{n+2m} = \{\zeta \in S^{n+2m} \mid \zeta_1 \ge \delta\}$. So, $m^h(\zeta) > 0$ for all $\zeta \in S_\delta^{n+2m}$ and from standard theory (see e.g. Debreu [6]) it follows that for every h the demand functions $x^h,\ s^h,\ z^h$ are continuous in ζ on S_δ^{n+2m}.

We now define the function $f\colon S_\delta^{n+2m} \to \mathbb{R}^{n+2m+1}$ by $f = (f^x, f^y, f^z, f^H, f^I)$, where

$$\begin{aligned} f^x(\zeta) &= \sum_{h \in H} (x^h(\zeta) - \omega^h) - x^0(\zeta) - x^1(\zeta), \\ f^y(\zeta) &= \sum_{i \in I} s^i(\zeta) - y^1, \end{aligned}$$

$$
\begin{aligned}
f^z(\zeta) &= \frac{z(\zeta)}{p_z}\left(\sum_{h\in H} p^h + \sum_{i\in I} t^i \frac{\partial q^i(z(\zeta))}{\partial z} - p_z\right), \\
f_h^H(\zeta) &= z^h(\zeta) - z(\zeta), \quad h \in H, \\
f_i^I(\zeta) &= s^i(\zeta) - q^i(z(\zeta)), \quad i \in I.
\end{aligned}
$$

Clearly, the functions f^x, f^y, f^H and f^I are well-defined and continuous because of the continuity of the demand and supply functions. Because of Assumptions 6.3 and 6.4 we have that $\frac{z(\zeta)}{p_z}$ and $\frac{\partial q^i(z(\zeta))}{\partial z}$ are also well-defined (bounded) and continuous and thus also f^z is well-defined and continuous at any $\zeta \in S_\delta^{n+2m}$. Therefore, the function f is a continuous function on S_δ^{n+2m}. Moreover the following lemma holds.

Lemma 6.5
For all $\zeta \in S_\delta^{n+2m}$ it holds that $\zeta^\top f(\zeta) \leq 0$.

Proof.
From the definition of $f(\zeta)$ it follows that

$$
\begin{aligned}
\zeta^\top f(\zeta) &= p^\top f^x(\zeta) + p_y f^y(\zeta) + p_z f^z(\zeta) + p^{H\top} f^H(\zeta) + t^{I\top} f^I(\zeta) \\
&= \textstyle\sum_{h\in H} p^\top (x^h(\zeta) - \omega^h) - p^\top x^0(\zeta) - p^\top x^1(\zeta) \\
&\quad + p_y(\textstyle\sum_{h\in H} s^h(\zeta) + p_y s^1(\zeta) - p_y y^1(\zeta) \\
&\quad + z(\zeta)\textstyle\sum_{h\in H} p^h + \sum_{i\in I} t^i \frac{\partial q^i(z(\zeta))}{\partial z} z(\zeta) - p_z z(\zeta)) \\
&\quad + \textstyle\sum_{h\in H} p^h z^h(\zeta) - z(\zeta)\sum_{h\in H} p^h \\
&\quad + \textstyle\sum_{h\in H} t^h s^h(\zeta) + t^1 s^1(\zeta) - \sum_{i\in I} t^i q^i(z(\zeta)) \\
&= \textstyle\sum_{h\in H}\left(p^\top x^h(\zeta) + (p_y + t^h)s^h(\zeta) + p^h z^h(\zeta) - p^\top \omega^h\right) \\
&\quad - (p^\top x^0(\zeta) + p_z z(\zeta)) \\
&\quad - (p^\top x^1(\zeta) - (p_y + t^1)s^1(\zeta) + p_y y^1(\zeta) \\
&\quad + \textstyle\sum_{i\in I} t^i \left(z(\zeta)\frac{\partial q^i(z(\zeta))}{\partial z} - q^i(z(\zeta))\right) \\
&\quad + z(\zeta)\textstyle\sum_{h\in H} p^h - z(\zeta)\sum_{h\in H} p^h \\
&\leq \textstyle\sum_{h\in H}(m^h(\zeta) - p^\top \omega^h) - (\pi^0(\zeta) + \pi^1(\zeta) + \pi(\zeta)) \\
&= 0.
\end{aligned}
$$

Q.E.D.

Observe that the inequality holds with equality when all budget constraints are satisfied with equality, which is true when none of the feasibility constraints in the restricted utility maximization problems are binding.

We now apply the next stationary point theorem, for a proof see for instance Van den Elzen [13].

Theorem 6.6
Let F be a continuous function from a convex and compact set $S \subset \mathbb{R}^k$ to $\mathbb{R}^k$. Then F has a stationary point $\zeta^* \in S$, i.e. there exists a point $\zeta^* \in S$, such that $\zeta^\top F(\zeta^*) \leq \zeta^{*\top} F(\zeta^*)$ for all $\zeta \in S$.

So, let ζ^* be a stationary point of the function f on S_δ^{n+2m}. Then we have the following lemma.

Lemma 6.7
Let ζ^* be a stationary point of f on S_δ^{n+2m}. Then $f(\zeta^*) \leq \underline{0}$.

Proof.
With Lemma 6.5 it follows that ζ^* satisfies

$$\zeta^\top f(\zeta^*) \leq \zeta^{*\top} f(\zeta^*) \leq 0, \text{ for all } \zeta \in S_\delta^{n+2m}. \tag{41}$$

Now, let $M = \max_{j=1}^{n+2m+1} f_j(\zeta^*)$ and $J = \{j \in \{1, \ldots, n+2m+1\} \mid f_j(\zeta^*) = M\}$. Suppose $M > 0$. First consider the case that $1 \in K$. Then it follows from inequality (41) that $\sum_{j \in J} \zeta_j^* = 1$ must hold and hence $\zeta_j^* = 0$ for $j \notin J$, implying that $\zeta^{*\top} f(\zeta^*) = M > 0$, which contradicts Lemma 6.5. In case $1 \notin J$, we must have that $\sum_{j \in J} \zeta_j^* = 1 - \delta$, $\zeta_1^* = \delta$ and $\zeta_j^* = 0$ for $j \notin J \cup \{1\}$. Since $x_1^h(\zeta) = A_1 > \omega_1$ when $p_1 = \delta$, it follows that $f_1^x(\zeta^*) > 0$, again contradicting $\zeta^{*\top} f(\zeta^*) \leq 0$. So, it follows that $M \leq 0$ and hence $f(\zeta^*) \leq \underline{0}$. Q.E.D.

We now prove the existence theorem.

Theorem 6.8 *Let $\mathcal{E} = \{T^0,\ (T^1, q^1),\ (u^h, q^h, \omega^h, \phi^{h0}, \phi^{h1}, \phi^h),\ h \in H\}$ be a private ownership economy satisfying Assumptions 6.1, 6.2, 6.3 and 6.4. Then there exists a Public Private Partnership Equilibrium.*

Proof.
We have shown already that under the assumptions there exists a stationary point $\zeta^* = (p^*, p_y^*, p_z^*, p^{H*}, t^{I*})$ of f in S_δ^{n+2m}. It remains to show that ζ^* with the corresponding profit and utility maximizing quantities satisfy the conditions of a PPPE. First, we show the market conditions. From Lemma 6.5 and Lemma 6.7 it follows that $f_j(\zeta^*) = 0$ if $\zeta_j^* > 0$. Since $\zeta_1^* = p_1^* \geq \delta > 0$, it follows that $f_1^x(\zeta^*) = 0$, i.e. the market of the first commodity is in equilibrium. Suppose $p_j^* = 0$ for some $j = 2, \ldots, n$. Then it follows from the monotonicity assumption and the restrictions in the utility maximizing problems that $x_j^h(\zeta^*) = A_j$, implying that $f_j^x(zeta^*) > 0$. Hence $p_j^* > 0$ for all j, and thus $f^x(\zeta^*) = \underline{0}$, which shows that all markets of the ordinary private commodities are in equilibrium. By the profit maximizing behavior of firm 1 it follows that $y^1(\zeta^*) = 0$ if $p_y^* = 0$, implying that

$f^y(\zeta^*) \geq 0$ if $p^*_y = 0$. Hence also $f_y(\zeta^*) = 0$, showing that the market of the complementary private commodity is in equilibrium. Analogously by the profit maximizing behavior of firm 0 it follows that $z(\zeta^*) = 0$ if $p^*_z = 0$, implying that $f^H_h(\zeta^*) \geq 0$ if $p^*_z = 0$ and thus also $f^H(\zeta^*) = \underline{0}$, showing that for each consumer the optimal level of infrastructure equals the production of infrastructure. Hence all the market clearing conditions are satisfied and thus none of the boundedness restrictions in the utility maximizing problems and profit maximizing problems are binding and so all consumers satisfy the unbounded utility maximizing and both producers satisfy the unbounded profit maximizing conditions.

From the properties of $f(\zeta^*)$ it also follows immediately that $t^{i*} = 0$ if $f^I_i(\zeta^*) = s^i(\zeta^*) - q^i(z(\zeta^*)) < 0$ and so also the condition that the tariff is zero when the quantity constraint on the use is non-binding is satisfied. To prove condition (xi) of Definition 4.1 we consider

$$f^z(\zeta^*) = \frac{z(\zeta^*)}{p^*_z}\left(\sum_{h \in H} p^{h*} + \sum_{i \in I} t^{i*}\frac{\partial q^i(z(\zeta^*))}{\partial z} - p^*_z\right) \leq 0.$$

First consider the case $z(\zeta^*) > 0$, implying that also $p^*_z > 0$ because of Assumption 6.2, part (i). Then $f^z(\zeta^*) = 0$ because of the properties of $f(\zeta^*)$ and hence

$$\frac{p^*_z}{z(\zeta^*)} f^z(\zeta^*) = \sum_{h \in H} p^{h*} + \sum_{i \in I} t^{i*}\frac{\partial q^i(z(\zeta^*))}{\partial z} - p^*_z = 0,$$

which shows condition (xi). In case $z(\zeta^*) = 0$ we have that $p^*_z = 0$ because of Assumption 6.2, part (iv). Then by Assumption 6.4 and the continuity of $z(\zeta)$ we have that $\frac{z(\zeta^*)}{p^*_z} = N(\zeta^*) > 0$. Hence

$$\frac{1}{N(\zeta^*)} f^z(\zeta^*) = \sum_{h \in H} p^{h*} + \sum_{i \in I} t^{i*}\frac{\partial q^i(z(\zeta^*))}{\partial z} - p^*_z \leq 0.$$

Since $p^*_z = 0$ and all other prices and tariffs are nonnegative, again the equation must hold with equality.

Finally, by definition $\pi^0(\zeta^*)$ and $\pi^1(\zeta^*)$ are the profit maximizing profits and as shown at the end of Section 4, also $\pi(\zeta^*)$ equals the equilibrium operator's profit. Hence also the consumers' incomes are correctly specified. Finally, from the first order utility maximization conditions (and firm 1's profit maximization condition) it follows that p^{H*} and t^{I*} satisfy the equilibrium conditions (i) and (ii) of Definition 4.1. Q.E.D.

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