Optimal Design of Trade Institutions*

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Abstract

We model an economy with social institutions that facilitate trade and induce three types of costs: establishment costs, access costs, and use costs. Use costs are specific transaction costs related to the use of these trade institutions. We assume that a trade institution is economically completely determined by the costs it imposes and by the effects on the trades it facilitates. We extend the Pareto efficiency concept to include various modes of organization of social institutions: the costs and benefits of these organizations are expressed in the trades they facilitate.

Within this setting we discuss a valuation equilibrium concept, in which all agents use a common conjectural price system that assigns to every trade institution the price vector that would prevail under it. This feature of the equilibrium is important in securing the second welfare theorem, and is new to the analysis of economies with costly trade. Since the use costs can be nonlinear, there are non-convexities that prevent the second welfare theorem from obtaining in a finite economy, but we show it for large economies.

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1 Designing trade institutions

What size and what kind of resources should be allotted to an institutional trade or market infrastructure? With the emergence of Internet trading, also known as "e-commerce," this is one of several fundamental open questions in contemporary economics. In fact, the trade infrastructure question is not addressed in the new institutional economics. (We refer to, e.g., Ménard, 1995.) In this paper we develop some notions that further our understanding of this important issue.

In our model we distinguish the following three functions in the design of a specific trade institution.

- The *provision system* offers a network of "plugs," which are economic agents, organizations, or admission points at which participating traders can get access to specific private commodities. On the Internet these plugs are the servers at which traders obtain access to web sites offering certain commodities. In a banking system the plugs refer to the Automatic Teller Machines (ATMs) available to access monetary instruments. In telephone service provision and electricity supply the "plug" concept has an obvious interpretation as well. We emphasize that the connections between the plugs in such a network are part of the provision system as well.

- The *connection process* refers to the searching process to obtain access to the provision system. This searching process is a costly action on the part of individual participants in the trade institution.

- The *user interface* refers to the frequency and intensity of the use of the provision system by a participant. This also refers to the interaction between suppliers and demanders to negotiate about the contracts and to overcome informational asymmetries.

From the above it should be clear that we understand an institutional trade infrastructure to be a structured configuration of collective projects[1] that facilitate the trade of private goods. This infrastructure may be concentrated around a physical network, such as a road system, an energy network, an "electronic highway," or a

[1] Here the notion of a *collective project* represents an indivisible public good that has widespread externalities. Such a collective project can be provided by a public government as well as a private club or corporation.

mall that facilitates transactions. It may also be a health care system that organizes the provision of private health services, or an organization (a club or a federation) that channels the provision of services to its members.

The trade institutions that make up a trade infrastructure can be provided *privately* and/or *publicly*. In each case, the trade institution retains its collective nature. Furthermore, a trade institution may be designed top-down, or it may emerge bottom-up. An example of top-down design is the regulation of markets for utilities, traditionally viewed as semi-publicly provided private goods, such as electricity, water, and telephone services. An example of bottom-up emergence is the rise of e-commerce. Top-down design commonly goes with public support and oversight of the institution, whereas bottom-up emergence usually is privately supported and unregulated.

In this paper we consider the design and implementation of certain trade institutions that facilitate the trade of a given set of certain private goods. We provide a general equilibrium framework in which one can study the *optimal* design of such trade institutions. In this respect the analysis presented in this paper is *purely normative*, and should not be construed to have any positive implications. In particular, we offer a system of valuing such trade institutional designs in relation to the services they provide. The insights reached are rather forceful: we establish the full decentralization of first best allocations using appropriate price and valuation systems, thus showing that certain institutional aspects in the trade process have to be taken into account fully in order to establish first best allocations. Thus, users determine by their independent, but price-coordinated, choices, how much they are willing to contribute towards the maintenance of the trade institutions. This is illustrated by the rise and decline of certain historical trade institutions such as the Dutch and English East Indian Companies and the medieval Hanse traders guild.

Next we address the precise nature of a trade institution from a general equilibrium perspective. When we buy a book from a bookstore, this activity easily fits into Debreu's (1959) contingent commodity concept: we consider the book as available at a given time, location, and state of nature, as a separate commodity. In this view, the traditional bookstore offers an extensive array of such commodities, and this array changes day by day. However, when we use an Internet meta-search to find a book from the cheapest on-line source, we are using a different bundling of commodities, as Debreu would view it: the ability to connect to the Internet, to the meta-search web site, to transact over the Internet securely, and so on. Thus, the experience of buying a good has many dimensions. In this paper, the institutional surroundings of a transaction have a new role. We would argue that this is a new and exciting concept of commodity, integral parts of which are information in various forms and the relationship of providers and demanders. Think of Debreu's concept of a commodity as including a new component: institutional design. For instance, having a medical procedure done by an HMO provider is in our model a different commodity than having it performed by an independent medical care provider.

The basic element of our model is the representation of these aspects of commodities through a differentiated notion of a trade institutional design. Thus, any complex combinations of aspects of a commodity in the new "e-commerce" world, for instance, are members of an unstructured set of potential designs of the trade institutions under consideration. Within the framework of this construction, it is relatively straightforward to encompass configurations of trading services viewed as relations between buyers, sellers, and other economic actors, such as auditors and marketeers. The amounts traded between buyers and sellers can then be measured by numbers, while the description of the trade institution itself captures their non-measurable aspects. This dichotomy in the description of commodities allows us to use powerful general equilibrium theory techniques while preserving the interesting complexity of the new world of e-commerce in our model.

The three functions in the design of a trade institution described above correspond to specific costs that are borne collectively or individually.

- *Setup costs* are the costs of establishing the provision system. In the case of e-commerce this covers the costs of installing telecommunications wires, the acquisition of servers, and the enforcement of electronic contracts. In principle, setup costs are borne collectively. (In practice we have observed a private takeover of the Internet after its publicly financed inception.)

- *Access costs* are the individually borne costs for a participating trader related to the connection process within the provision system. These costs are independent of quantities traded. In the case of e-commerce these costs cover the purchase of a modem, the use of telephone services to gain access to the Internet, and the opportunity costs of waiting time to access web sites of one's choice.

- *Use costs* are the transaction costs related to obtaining a specific commodity provided. These costs refer to the use of the user interface including the costs of searching, negotiating and overcoming informational asymmetries. These costs depend explicitly on the quantities traded.

Access costs are relatively easy to measure and to make accountable to the participating agents. In many situations, costs are attributed to agents only on the basis of access, for example, local telephone calls in the USA. On the other hand, some providers may choose to bear these costs and to provide free access, in order to reap the external effects of scale of the provider system.

Use costs are distinguished because they are closest to what is known as "transaction costs" in the literature. We prefer the term use costs to make a sharp distinction between general transaction costs of the overall trade infrastructure in the economy and the transaction costs related to the specific trade institutions under consideration. The problem with use costs, however, is that they are intrinsically related to the benefits an agent receives from participating in a trade institution, so they cause non-convexities in individual utility functions that are hard to overcome. We resolve this problem in our second welfare theorem by invoking a large economy.

In the new institutional economics literature, markets are understood to be goods themselves. Moreover, "the institutions of a market provide an oriented set of complementary immaterial capital goods which reduces the direct cost of individual transactions." (Loasby, 2000, page 300.) Our approach attempts to directly merge the choices made with regard to these market capital goods as collective trade infrastructural projects into the standard Walrasian model of a competitive market system. The three types of costs related to these collective trade infrastructural goods represent the different cost configurations that are imposed on the society and the market participants when certain choices are made. This allows us indeed to consider changes — in particular reductions — of the corresponding costs of making transactions.

The nature of the trade institution in our model is multifaceted and wide-ranging. Its specification is selected from a set of possible designs. A design of the trade institution facilitates access to and trade of a finite number of given private commodities. The trade institution may supply books, CD's and electronics on the Internet; financial services in a banking network of ATMs; a network of medical specialists in a hospital system providing private medical services; or any combination of the above. Apart from the finite number of private goods traded within the trade institution, our model assumes one composite private commodity, which incorporates the other commodities in the economy. The model is completed with a production technology that describes how the composite commodity is converted into the commodities traded within the trade institution considered. It also allows us to use Pareto efficiency as the optimality criterion in the presence of a trade institution.

The equilibrium concept used to analyze behavior in our model is that of *valuation equilibrium*. We assume that all private commodities traded within the trade institution and the composite commodity are traded on competitive markets, so a uniform market price can be assigned to each of them. The exact nature of each commodity traded within the trade institution, however, is determined by, and varies with, the design of the trade institution. Hence, there exists a trade-off between the valuation of a specific trade institutional design and the value or price of each private commodity. The valuation equilibrium concept captures the decentralization of individual decisions in the determination of the design of the trade institution.

This implies that a valuation equilibrium is an allocation, a production plan and, *for every* specific design of the trade institution, a price vector for private commodities *as well as* a tax and subsidy system — akin to Lindahl personalized prices — such that, given the price and tax-subsidy systems, (i) each agent maximizes her/his utility function over her/his budget set; (ii) the production plan of the commodities traded maximizes profit; and (iii) the selected trade institutional design maximizes the social surplus. We show that all valuation equilibria are Pareto efficient, extending the first welfare theorem to our model. Regarding the second welfare theorem, we show that in large economies Pareto efficient allocations can be decentralized using appropriate price and valuation systems.

Our approach allows for the study of certain specifications of trade infrastructural institutions. For example, the public-private partnership equilibrium discussed in van der Laan, Ruys and Talman (2000), which provides a way to allocate the setup costs of the trade infrastructure to its users, is shown to be efficient. Therefore, it is a specification of the model presented here.

Alternative approaches to the topic of this paper mostly use techniques from game theory and mechanism design theory. The literature has until now mostly considered certain aspects of trade infrastructures separately. For example, Baesemann (1977) studies the formation of small market places in a dynamic model, Bose and Pingle (1995) discuss the endogenous formation of stores as intermediary traders, and Yang and Ng (1993) develop a number of highly specified general equilibrium models in which the division of labor and the extent of markets are endogenous. (See also Yang 2001.) Shubik and his collaborators have studied a class of "trading-post" models, which are static models of exchange in separate markets for each commodity, with traders acting non-cooperatively; the main results of this work concern the inefficiency of Nash equilibria of these trading games and convergence of Nash equilibria to competitive equilibria under replication; see, for example, Shubik (1984). Furthermore, Berliant and Wang (1993) develop a general equilibrium model in which a linear city emerges endogenously, where with high market set-up costs, a unique central business district is the outcome. A more comprehensive approach is developed in Berliant and Konishi (2000), who discuss a general equilibrium model with mobile consumers and immobile technologies in which the exchange of commodities occurs at marketplaces, the number and location of which is determined exogenously. Finally, Zhou, Sun and Yang (1999) develop a general equilibrium model with specialization and division of labor, which includes transaction costs and allows for increasing returns. Working with an atomless economy and personalized production sets, they show existence of equilibrium, the two welfare theorems, and core equivalence.

2 The structure of the economy

Throughout this paper we use the symbol A to indicate the set of all economic agents. The set A can be finite or infinite. We assume that if A is infinite, it is endowed with a locally compact Hausdorff topology $\mathcal{T}$. Denote by Σ the σ-algebra of Borel sets generated by the topology $\mathcal{T}$ and denote by $\mu : \Sigma \to [0, 1]$ a regular Borel measure on (A, Σ) such that $\mu(A) = 1$, i.e., μ is a probability measure. If A is finite, we assume that $\Sigma = \{E \mid E \subset A\}$.

There is one *composite commodity* in the economy with index 0. It represents those commodities that are to be traded through well established markets.[2]

There are ℓ *trade commodities* indexed by the set $L = \{1, \ldots, \ell\}$ that are traded within a newly designed trade institution. As mentioned in the introduction the trade institution provides access to these trade commodities and facilitates the trade of these commodities.

The set of all commodities in the economy is denoted by $M = \{0\} \cup L$, where M

[2]We remark that all the results in this paper remain valid if we were to replace commodity 0 with a finite number of commodities. We avoid this to keep the notation relatively simple.

contains $m = \ell + 1$ commodities. The consumption space is given by the nonnegative orthant of the m-dimensional Euclidean space $\mathbb{R}^m_+$. We indicate by an integrable function $w : A \to \mathbb{R}_{++} \times \{0\}^\ell$ the *endowment* of (private) commodities attributed to the agents in the economy. Agents have zero endowment of the trade commodities provided through the newly designed trade institution, but they have a positive endowment of the composite commodity.

The trade commodities are produced according to a production set $Y \subset \mathbb{R}^m$, which is assumed to be a nonempty, convex pointed cone, comprehensive from below (free disposal), and such that $Y \cap \mathbb{R}^m_+ = \{0\}$. This amounts to the assumption of constant returns to scale.[3] In conjunction with the assumption on endowments, it follows that there is only one primal input, the composite commodity.

Next we turn to the description of the trade institution itself. The trade institution can have several *designs*, determined by the specification of the provision system, the design of the connection process, and the user interface introduced in the previous section. Thus, the design of the trade institution is selected from a set Γ of all possible specifications. For each design specification $\gamma \in \Gamma$ we can now describe its different constitutional aspects by the costs related to them.

First, the provision system is represented through the setup costs, which are given by a vector of fixed inputs $c(\gamma) \in \mathbb{R}^m_+$. These costs are related to the construction and/or maintenance of the provision system. As discussed before these costs are assumed to be independent of the quantities traded. (Note that there is an implicit connection between the cost and the capacity of the trade institution, which is incorporated as a feature of γ.) Thus, we introduce $c : \Gamma \to \mathbb{R}^m_+$ as the *setup cost function* assigning to each design $\gamma \in \Gamma$ the fixed input vector $c(\gamma) \in \mathbb{R}^m_+$.[4]

The second component of the trade institution is the *access network*. Before an agent can use or consume private commodities, she or he has to access the provision system. The costs of making these connections are called access costs and these costs are borne by individual agents. For each design $\gamma \in \Gamma$ the induced access costs are represented by a function $r_\gamma : A \to \mathbb{R}^m_+$, called the *access cost function* related to

[3] We easily may replace this constant returns to scale hypothesis with an assumption of decreasing returns to scale, but it would not offer additional insight into the model. However, it would unnecessarily complicate the notation.

[4] This formulation may appear to rule out substitutability of inputs. However, we could have specified c as a correspondence to deal with this issue without any substantial impact on the analysis except for a notational complication — see Diamantaras, Gilles and Scotchmer (1996).

$\gamma \in \Gamma$.

This formulation of access costs is very general. It allows these costs to be in terms of the composite and/or any of the traded commodities. As a special case, it allows the access costs to be only in terms of the composite commodity. However, the additional generality of allowing the access costs to be in terms of the traded commodities also is useful in capturing some realistic situations. For example, one has to use the Internet to get appropriately configured browser software in order to be able to do online shopping or banking. This is a true up-front cost for trading on the Internet.

The third component of the trade institution is the *user interface*, which refers to the relation between the provision system and the consumer of the commodities provided. The costs related to interfacing are usually called "transaction costs." Here these costs only refer to the costs of trading the ℓ specific trade commodities, not all commodities, some of which are the commodities subsumed in commodity 0. The latter are taken as given — as part of a settled trade infrastructure for the composite commodity — and as such a feature which is not subject to change in our framework. Therefore the user interface costs are not a general form, but rather a specific form of (total) transaction costs. We refer to these costs as the *use costs*. In principle these use costs depend on the characteristics of the provision system and the user, and on the amount of the commodities traded.[5] Since the agents hold no endowments of the commodities subject to these use costs, the use costs only depend on the quantities actually consumed.

Within our model there is a clear distinction between access and use costs. Indeed, in our model, individuals always incur access costs; they do so to learn about the trade institution and to get an idea of what is available for trading. This implies that we assume that agents make these access costs to get acquainted with the trade institution. However, after this learning process agents may opt to stay home and not trade. In that case they do not incur any use costs.[6]

Formally, we introduce for each design $\gamma \in \Gamma$ a *use cost function* $t_\gamma : A \times \mathbb{R}^m_+ \to \mathbb{R}^m_+$ such that, for every $a \in A$, every $x_0, x'_0 \in \mathbb{R}_+$, and every $x_1 \in \mathbb{R}^\ell_+$, we have $t_\gamma(a, x_0, x_1) = t_\gamma(a, x'_0, x_1)$. If $a \in A$ is an agent and $x_1 \in \mathbb{R}^\ell_+$ is her final consumption

[5]This is a standard assumption in the classical transaction costs general equilibrium literature. We also refer to Foley (1970), Kurz (1974), Heller and Starr (1976), and Zhou, Sun and Yang (1999).

[6]We remark that in other models, individual agents have the option of also not incurring access costs. In particular we refer to Gilles and Diamantaras (2003).

bundle of traded commodities, then $t_\gamma(a, x_0, x_1) \in \mathbb{R}^m_+$ denotes the bundle of all commodities, including the composite commodity 0, lost due to costs of transacting through the trade institution of design γ. This cost is independent of the amount of the composite commodity x_0.

It is impossible to give a fully linear specification of use costs. This is due to the special role assigned to the composite commodity. Commodity 0 is namely not subject to the use cost function. The next example explores some alternative formulations that introduce partial linearity of the use cost function.

Example 2.1 There are different ways to introduce an element of linearity in use costs. We discuss three possibilities. Let $a \in A$ and $(x_0, x_1) \in \mathbb{R}^m_+$.

Firstly, the use costs are linear in the vector of the ℓ traded commodities and do not involve the composite commodity. Let $\lambda \in [0, 1]$ and set $t^\lambda_\gamma(a, x_0, x_1) = (0, \lambda x_1) \in \mathbb{R}^m_+$.

Secondly, the use costs do not involve the ℓ traded commodities but only the composite commodity as a fraction of the length of the traded commodity vector. Let $\mu \in [0, 1]$ and set $t^\mu_\gamma(a, x_0, x_1) = (\mu\|x_1\|, 0) \in \mathbb{R}^m_+$.

Thirdly, the use costs are a combination of the two previous cases. Let $\nu \in [0, 1]$ and set $t^\nu_\gamma(a, x_0, x_1) = (\nu\|x_1\|, \nu x_1) \in \mathbb{R}^m_+$. □

The definition of use costs allows us to distinguish gross consumption from net or final consumption. For this purpose let $x \in \mathbb{R}^m_+$ be the final consumption bundle of agent a and $t_\gamma(a, x)$ the (general) use costs associated with design γ. We define the function $g_\gamma : A \times \mathbb{R}^m_+ \to \mathbb{R}^m_+$ by $g_\gamma(a, x) = x + t_\gamma(a, x)$ and call it the *gross consumption function*. Note that from the above $g_\gamma(a, 0) = 0$ for every $a \in A$ and $\gamma \in \Gamma$.

We do not assume that a trade institution induces any externalities other than the three types of costs related to its establishment and its use introduced above. Thus, there are no direct externalities in the sense that agents do not take into account the design of a trade institution directly into their utility functions. For each agent $a \in A$ we represent a's preferences by a utility function $U_a : \mathbb{R}^m_+ \to \mathbb{R}$, which depends only on the (net) quantities of the private goods consumed by that agent.[7]

We apply the following conventions. Let $a \in A$. We call the utility function $U_a : \mathbb{R}^m_+ \to \mathbb{R}$ *monotone* if for all $x, y \in \mathbb{R}^m_+$, $x \gg y$ implies $U_a(x) > U_a(y)$ and *strictly*

[7]We can incorporate without much additional complication direct externalities regarding the trade institution established in the economy. We refer to Diamantaras and Gilles (1996) and Diamantaras, Gilles and Scotchmer (1996) for details.

monotone if for all $x, y \in \mathbb{R}^m_+$, $x > y$ implies $U_a(x) > U_a(y)$, where we use the vector inequalities $\gg$, $>$, and $\geqq$.

Summarizing, we define an economy as follows:

Definition 2.2 *A tuple* $\mathbb{E} := \langle (A, \Sigma, \mu), w, \{U_a\}_{a\in A}, Y, (\Gamma, c, r, t)\rangle$ *is an* ***economy*** *if for every design* $\gamma \in \Gamma$, $\{U_a\}_{a\in A}$ *and* w *are jointly measurable in the sense of Hildenbrand (1974) and the corresponding cost functions* r_γ *and* t_γ *are integrable on* A *and* t_γ *is continuous on* $\mathbb{R}^m_+$.

The concept of an allocation in this economy depends on the specification of the trade institutional design. Initial resources cannot be spent only on consumption, but also have to be spent on the maintenance of the trade institution itself. This is captured in our notion of feasibility.

Definition 2.3 *An* ***allocation*** *in* $\mathbb{E}$ *is a triple* (γ, f, y) *where* $\gamma \in \Gamma$ *is a design of the trade institution,* $f\colon A \to \mathbb{R}^m_+$ *is an integrable distribution of commodities for final consumption, and* $y \in Y$ *is a net production vector of the trade commodities. An allocation* (γ, f, y) *is* ***feasible*** *if*

$$\int f\, d\mu + \int t_\gamma(a, f(a))\, d\mu(a) + \int r_\gamma\, d\mu + c(\gamma) \leqq \int w\, d\mu + y. \tag{1}$$

A feasible allocation (γ, f, y) *is* ***Pareto efficient*** *in* $\mathbb{E}$ *if there is no feasible allocation* (δ, g, z) *such that for almost every agent* $a \in A:\ U_a(g(a)) \geqq U_a(f(a))$ *and there is a nonnegligible set* $E \in \Sigma$ *with* $U_b(g(b)) > U_b(f(b))$ *for all* $b \in E$.

We complete the initial development of the model with the introduction of the following hypothesis.

Axiom 2.4 *For every design* $\gamma \in \Gamma$, *there exists a production plan* $y \in Y$ *such that* $\int r_\gamma\, d\mu + c(\gamma) \ll \int w\, d\mu + y$.

Together with the continuity of the use cost function and the hypothesis that zero trade does not generate any use costs, Axiom 2.4 implies that there exists a feasible allocation that assigns to each agent in a non-negligible set a non-zero consumption bundle.

3 Equilibrium concepts and results

We impose certain operational properties on use cost functions in order to allow for a decentralized organization of decisions leading to a Pareto efficient allocations. The next definition states the two most important properties.

Definition 3.1 *Let* $\mathbb{E}$ *be an economy.*

(a) *For design* $\gamma \in \Gamma$ *the use cost function* t_γ *is said to be* ***monotone*** *if it is component-wise nondecreasing, i.e., if for all bundles* $x, y \in \mathbb{R}^m_+$ *and* $i \in M$: $x^i \geqq y^i$ *implies that* $t^i_\gamma(a, x) \geqq t^i_\gamma(a, y)$ *for every agent* $a \in A$.
$\mathbb{E}$ *is said to* ***exhibit monotone use costs*** *if for every design* $\gamma \in \Gamma$ *the use cost function* t_γ *is monotone.*

(b) $\mathbb{E}$ *is said to* ***exhibit bounded use costs*** *if there exists a number* $K > 0$ *such that for every design* $\gamma \in \Gamma$*, every bundle* $x \in \mathbb{R}^m_+$*, and every agent* $a \in A$ *it holds that* $t_\gamma(a, x) \leqq K\, x$.

Next we address the question of decentralization of (first-best) Pareto efficient allocations. By means of an economy $\mathbb{E}_g$ which is derived from $\mathbb{E}$ and in which the use costs are internalized into the utility function, we are able to decentralize Pareto efficient allocations quite straightforwardly. Since the definitions in $\mathbb{E}_g$ have suitable counterparts in $\mathbb{E}$, we thus arrive at the desired decentralization of Pareto efficient allocations.

Axiom 3.2 (Order preservation hypothesis) *For every potential design* $\gamma \in \Gamma$ *the gross consumption function* g_γ *is injective on* $\mathbb{R}^m_+$.

Axiom 3.2 is crucial in the further construction of the model. We claim that it is not very restrictive. An example not satisfying the order preservation hypothesis follows. This example shows that only a rather specific, relatively irrelevant class of use cost functions fails to satisfy this hypothesis. The point of this hypothesis — and, by extension, of the analysis of this section — is to delineate the class of economies in which first-best design of trade institutions is attainable under non-trivial use costs. Further research has to make clear whether this hypothesis can be weakened.

Example 3.3 Consider an situation where $\ell = 2$ and an agent $a \in A$ such that $t_\gamma(a, x_0, x_1, x_2) = (0, x_2, x_1)$. It follows that $g_\gamma(a, x_0, x_1, x_2) = (x_0, x_1 + x_2, x_1 + x_2)$, which is not an injective (one-to-one) function. This form of the use cost function, in which trading one good uses up another good, seems to be the most basic functional form for which Axiom 3.2 fails. Obviously, we exclude such use cost functions. On the other hand, this class of use cost functions seems less relevant for the purpose on hand, in particular because use cost structures discussed in Example 2.1 satisfy the order preservation hypothesis. □

Axiom 3.2 implies that g_γ has an inverse. This inverse of $g_\gamma(a, \cdot)$ is the function $n_\gamma : A \times \mathbb{R}^m_+ \to \mathbb{R}^m_+$ defined by $n_\gamma(a, g_\gamma(a, x)) = x$. So, if agent $a \in A$ obtains a bundle $y \in \mathbb{R}^m_+$ in the market, he actually consumes a quantity given by $n_\gamma(a, y) \in \mathbb{R}^m_+$. The function n_γ is called the *net consumption function* derived from the use cost function t_γ. The following lemma summarizes the properties of the net consumption function. For its proof we refer to Appendix A.

Lemma 3.4 *Let $\gamma \in \Gamma$. Then:*

(i) *$n_\gamma(\cdot, x)$ is integrable on (A, Σ, μ) for all $x \in \mathbb{R}^m_+$, and $n_\gamma(a, \cdot)$ is continuous for all $a \in A$.*

(ii) *For every $a \in A$ and $x \in \mathbb{R}^m_+$: $n_\gamma(a, x) \leqq x$.*

(iii) *For every $a \in A$ and every bundle $x \in \mathbb{R}^m_+$ there exists a bundle $y \in \mathbb{R}^m_+$ with $n_\gamma(a, y) \gg x$.*

(iv) *If t_γ is monotone, then for every $a \in A$ and for all bundles $x, y \in \mathbb{R}^m_+$: $x \geqq y$ ($x > y$) implies that $n_\gamma(a, x) \geqq n_\gamma(a, y)$ ($n_\gamma(a, x) > n_\gamma(a, y)$).*

In the sequel we usually assume that use cost functions are monotone, and, thus, all properties (i)–(iv) in Lemma 3.4 are satisfied.

Example 3.5 Consider the first case discussed in Example 2.1. For the given use cost function t^λ_γ it can easily be derived that for every $a \in A$ and $(x_0, x_1) \in \mathbb{R}^m_+$ the net consumption function is given by

$$n^\lambda_\gamma(a, x_0, x_1) = \left(0, \frac{1}{1 + \lambda} x_1\right).$$

This implies that there is no fully linear relation between the net and gross consumption vectors. Remark also that this net consumption function satisfies all properties of Lemma 3.4. □

The *gross utility function,* denoted by $\bar{U}_a : \mathbb{R}^m_+ \times \Gamma \to \mathbb{R}$, is defined by $\bar{U}_a(f,\gamma) = U_a(n_\gamma(a,f))$ for every $f \in \mathbb{R}^m_+$. It internalizes both the benefits and the costs of the interaction between provider and user. The next example shows that nonlinear use costs functions can lead to non-convexities of the gross utility function, even though the (original) utility function satisfies all regularity requirements.

Example 3.6 Let $\ell = 1$ and denote the composite good by x and the traded good by y. Choose any $a \in A$, $x \in \mathbb{R}_+$, and $y \in \mathbb{R}_+$, and set $t_\gamma(a,x,y) = \left(0, \frac{1}{2}\sqrt{y}\sqrt{x+y} - \frac{1}{2}y\right)$. This implies that

$$g_\gamma(a,x,y) = (x,y) + t_\gamma(a,x,y) = \left(x, \tfrac{1}{2}y + \tfrac{1}{2}\sqrt{y}\sqrt{x+y}\right).$$

Observe that $n_\gamma(a,x,y) = \left(x, y - 1 + \frac{y}{y+1}\right)$. Consider a linear utility function $U_a(x,y) = x + y$. Then the gross utility function is given by:

$$\bar{U}_a(x,y) = U_a(n_\gamma(a,x,y)) = x + y - 1 + \frac{y}{y+1}.$$

This utility function represents non-convex preferences. □

The discussion on the gross representation is summarized as follows:

Definition 3.7 *The tuple* $\mathbb{E}_g := \langle (A,\Sigma,\mu), w, \{\bar{U}_a\}_{a\in A}, (\Gamma,c,r), Y\rangle$ *is the* ***gross representation*** *of the economy* $\mathbb{E} = \langle (A,\Sigma,\mu), w, \{U_a\}_{a\in A}, (\Gamma,c,r,t), Y\rangle$ *if for every potential design* $\gamma \in \Gamma$ *and every agent* $a \in A$ *the gross utility function* $\bar{U}_a : \mathbb{R}^m_+ \times \Gamma \to \mathbb{R}$ *is given by* $\bar{U}_a(\cdot,\gamma) = U_a(n_\gamma(a,\cdot))$.

We emphasize that the gross representation $\mathbb{E}_g$ of an economy $\mathbb{E}$ incorporates the use costs into the utility functions, but does not affect the representation of the setup and access costs. Hence, the transition to the gross representation of some economy incorporates the consumption and service technology into the preference structure.[8] An alternative formulation would be to incorporate access costs as well as use costs

[8]For well established trade technologies this is evidently a natural occurrence: The gas used to drive one's car to and from downtown or the mall for shopping is not considered separately as a use cost, but rather incorporated directly into the agent's preference relation.

into the gross representation. Our reason for not doing this is that access costs are based on a deliberate act of accessing γ, whereas the use costs are related to the nature of the facilities that γ provides.

Next we convert all definitions as employed in $\mathbb{E}$ to their appropriate counterparts in the gross representation $\mathbb{E}_g$. An allocation (γ, f, y) is redefined in the sense that $f : A \to \mathbb{R}^m_+$ is an integrable distribution of private goods for *gross consumption*, i.e., *before* subtraction of use costs. An allocation (γ, f, y) is *feasible* in $\mathbb{E}_g$ if

$$\int f \, d\mu + \int r_\gamma \, d\mu + c(\gamma) \leqq \int w \, d\mu + y. \tag{2}$$

It is clear that (2) is equivalent to (1). Since use costs depend on the trade institution design chosen, the refinement of the notion of an allocation causes a redefinition of the efficiency concept. A feasible allocation (γ, f, y) in $\mathbb{E}_g$ is *Pareto efficient* in $\mathbb{E}_g$ if there is no other feasible allocation (δ, h, z) in $\mathbb{E}_g$ such that for almost every agent $a \in A$, $\bar{U}_a(h(a), \delta) \geqq \bar{U}_a(f(a), \gamma)$, and there exists some nonnegligible coalition $E \in \Sigma$, $\mu(E) > 0$, such that for every $b \in E$: $\bar{U}_b(h(b), \delta) > \bar{U}_b(f(b), \gamma)$. The proof of the following lemma is trivial and therefore omitted.

Lemma 3.8 *Let $\mathbb{E}_g$ be the gross representation of $\mathbb{E}$. If an allocation (γ, f, y) in $\mathbb{E}$ is Pareto efficient, then the corresponding gross allocation $\left(\gamma, \left(g_\gamma\left(a, f(a)\right)\right)_{a\in A}, y\right)$ is Pareto efficient in $\mathbb{E}_g$. If an allocation (γ, f, y) in $\mathbb{E}_g$ is Pareto efficient, then the corresponding net allocation $\left(\gamma, \left(n_\gamma\left(a, f(a)\right)\right)_{a\in A}, y\right)$ is Pareto efficient in $\mathbb{E}$.*

We are interested in the question whether Pareto efficient allocations can be decentralized through some appropriate price system. We introduce

$$\Delta := \left\{ p \in \mathbb{R}^m_+ \,\middle|\, \sum_{i=1}^m p_i = 1 \right\}$$

as the simplex of all normalized price vectors.

Definition 3.9 *A feasible allocation (γ, f, y) is a **valuation equilibrium** in $\mathbb{E}_g$ if there exist a price system $p : \Gamma \to \Delta$, a production plan $\phi : \Gamma \to Y$ with $\phi(\gamma) = y$, and a valuation system $V : A \times \Gamma \to \mathbb{R}$ such that*

(i) *for every $\delta \in \Gamma$ and every $z \in Y$, $0 = p(\delta) \cdot \phi(\delta) \geqq p(\delta) \cdot z$,*

(ii) *for every* $\delta \in \Gamma$: $V(\cdot, \delta)$ *is integrable and for almost every agent* $a \in A$: $V(a, \delta) \leqq p(\delta) \cdot w(a) - p(\delta) \cdot r_\delta(a)$,

(iii) *in equilibrium there is budget balance, i.e.,* $\int V(\cdot, \gamma)\, d\mu = p(\gamma) \cdot c(\gamma)$,

(iv) γ *minimizes the deficit* $p(\delta) \cdot c(\delta) - \int V(\cdot, \delta)\, d\mu$ *over all* $\delta \in \Gamma$, *and*

(v) *for almost every agent* a *in* A, $(f(a), \gamma)$ *maximizes the gross utility function* $\bar{U}_a$ *on the budget set*

$$\left\{ (h, \delta) \in \mathbb{R}^m_+ \times \Gamma \,\middle|\, p(\delta) \cdot h + p(\delta) \cdot r_\delta(a) + V(a, \delta) \leqq p(\delta) \cdot w(a) \right\}.$$

The notion of valuation equilibrium encompasses the requirement that all agents have coordinated expectations in the sense that the price system p, the valuation system V as well as the production plan ϕ are *conjectural*. Conjectural price systems were introduced by Diamantaras and Gilles (1996).[9] Using these price systems implies that agents have common knowledge about the price and production changes that occur when an alternative trade institutional design is implemented. Although this requirement seems very demanding, Diamantaras, Gilles and Scotchmer (1996) show that it is a necessary condition for the second welfare theorem.

The next result is the first welfare theorem for our model. For a proof of this result we refer to Appendix B.

Theorem 3.10 *Let* $\mathbb{E}$ *be an economy exhibiting monotone use costs such that for all agents* $a \in A$ *the utility function* U_a *is monotone. Then for each valuation equilibrium* (γ, f, y) *in* $\mathbb{E}_g$ *its net representation* $\left(\gamma, \left(n_\gamma\left(a, f(a)\right)\right)_{a \in A}, y\right)$ *is Pareto efficient in* $\mathbb{E}$.

We remark that it is possible to formulate valuation equilibrium for the original economy $\mathbb{E}$ by changing (v) in the definition of valuation equilibrium to refer to the

[9] Mas-Colell (1980) seminally introduced the notion of valuation equilibrium for economies with collective goods, which he called public projects, and one private good. Diamantaras and Gilles (1996) and Diamantaras, Gilles, and Scotchmer (1996) offer generalizations of valuation equilibrium to accommodate multiple private goods and introduced conjectural price systems. Hammond and Villar (1999) developed a similar modification of the valuation equilibrium concept which does not require agents to use conjectural price systems. Their construction, however, relies on a central authority that can threaten the destruction of resources out of equilibrium. Our concept does not require this; in fact, it requires that the central authority always plan on a deficit out of equilibrium, with the deficit equal to zero at equilibrium.

net utility function, and by using the original feasibility condition for allocations, (1). For this formulation the first welfare theorem goes through without imposing Axiom 3.2 and without significant changes in its proof.

The converse relationship, usually known as the second welfare theorem, can only be achieved under certain additional requirements. The reasons are: (i) in general, nonlinear use costs may cause non-convexities and thus failure of the decentralization of Pareto efficient allocations (see Example 3.6); (ii) we have to avoid giving some agents zero income. In order to circumvent these issues we consider an atomless economy and distinguish between equilibrium and quasi-equilibrium.

Definition 3.11 *A feasible allocation* (γ, f, y) *is a* ***valuation quasi-equilibrium*** *in* $\mathbb{E}_g$ *if it satisfies conditions (i)–(iv) of Definition 3.9 and the following property:*

(v') *For almost every agent* a *in* A*,*

$$p(\gamma) \cdot f(a) + p(\gamma) \cdot r_\gamma(a) + V(a, \gamma) = p(\gamma) \cdot w(a)$$

and for every $(h, \delta) \in \mathbb{R}_+^m \times \Gamma$ *with* $\bar{U}_a(h, \delta) > \bar{U}_a(f(a), \gamma)$*:*

$$p(\delta) \cdot h + p(\delta) \cdot r_\delta(a) + V(a, \delta) \geqq p(\delta) \cdot w(a).$$

Finally, we say that a utility function $U : \mathbb{R}_+^m \to \mathbb{R}$ has *compact lower contour sets* if for every $\hat{x} \in \mathbb{R}_+^m$ the set $\{x \in \mathbb{R}_+^m \mid U(x) \leqq U(\hat{x})\}$ is compact in the Euclidean topology. For a proof of Theorem 3.12 we refer to Appendix C.

Theorem 3.12 *Let* $\mathbb{E}$ *be an economy exhibiting bounded and monotone use costs such that for every agent* $a \in A$ *the utility function* U_a *is continuous and strictly monotone. Then the following statements hold:*

(a) *If* (A, Σ, μ) *is atomless, then each Pareto efficient allocation in* $\mathbb{E}$ *can be supported as a valuation quasi-equilibrium in* $\mathbb{E}_g$*.*

(b) *Additionally, assume that* U_a*,* $a \in A$*, have compact lower contour sets. If* (A, Σ, μ) *is atomless, then each Pareto efficient allocation in* $\mathbb{E}$ *can be supported as a valuation equilibrium with strictly positive prices in* $\mathbb{E}_g$*.*

Theorem 3.12 shows that conjectural price systems and valuations achieve Pareto optimality in an economy with a trade institution under perfectly competitive behavior. Thus, all agents are required to possess abilities to compute and anticipate price changes resulting from the selection of an alternative design of the trade institution.

4 Concluding remarks

Throughout this paper we have mainly formulated the normative concept of Pareto optimality within the setting of the economy $\mathbb{E}$. On the other hand, we have introduced all equilibrium concepts based on coordination through a price mechanism in its gross representation $\mathbb{E}_g$, incorporating the use costs into the agents' preferences. This has a natural interpretation. Indeed, Loasby (2000, page 306) refers to this as the "second function" of market organization: "by allowing us to cope with transactions it frees our cognitive powers for other uses." This is precisely represented by the embedding of use costs into the preferences. While using the price mechanism, agents incorporate these costs related to the use of trade technologies conveniently into their consumption decisions, i.e., through optimization of a modified preference relation over their budget set. On the other hand, while considering normative efficiency concepts, it seems natural to consider all costs — also those related to the use of certain trade technologies — *explicitly* rather than *implicitly*. This is exactly the case with the original "net" representation $\mathbb{E}$.

We can avoid going back and forth between the gross and net representations, if we assume that *all* commodities, including the composite commodity, are traded via the trade institution and impose linear use costs. Contrary to the cases discussed in Example 2.1, in this case it is possible to formulate truly linear use costs. This case has been developed further in Gilles, Diamantaras and Ruys (1994).

Future directions for research include the extension of the current model to include a reconciliation of our model with other models of economies with transaction costs, for example, the model of Zhou, Sun and Yang (1999). Also, applications of our approach include positive studies of more specified models of the selection of trade institutions. Here we are referring to the role of Axiom 3.2 in the current model and the question whether we can weaken this requirement.

Most importantly, our analysis extends to the organization of a "service" economy, which pertains to about seventy percent of GDP in advanced economies. A service generalizes upon the standard Debreu-inspired concept of a commodity, in that it is carried by a relation rather than by a physical unit that can be measured and represented into a Euclidean space. Our approach requires us to go beyond the regular Euclidean framework and use the fundamental notion of a non-measurable collective project. For more details see Ruys(2002).

Finally, by making Γ multidimensional, we could develop models in which specific aspects of the trade institution are priced separately. For example, we could then determine the correct valuation for a Parisian to access the Minitel separately from accessing the Internet.[10]

Our work can be criticized for "only" being normative and for not mentioning the monopoly power that market-makers have in practice. We feel that there is room for a normative study of institutions for trading, not least so as to serve as a benchmark against which more detailed, game-theoretic models may be compared. Regarding monopoly power, we have assumed that the markets for the private commodities are competitive. The details of organization of an institution γ are purposefully left in the abstract. Details of the governance structure of the industry are part of the definition of γ, as we have implicitly suggested in the introduction and explicitly discussed in the previous two paragraphs. The monopoly rents possibly received by market-makers could naturally be integrated into the setup cost $c(\gamma)$ of γ. Viewed from this angle, our model provides a capacious framework within which first- and second-best analysis of specific economies could be conducted.

References

[1] Baesemann, R.C. (1977), "The Formation of Small Market Places in a Competitive Economic Process — the Dynamics of Agglomeration", *Econometrica,* 45, 361–374.

[2] Berliant, M., and P. Wang (1993), "Endogenous Formation of a City Without Agglomerative Externalities or Market Imperfections: Marketplaces in a regional economy", *Regional Science and Urban Economics,* 23, 121–144.

[3] Berliant, M., and H. Konishi (2000), "The Endogenous Formation of a City: Population Agglomeration and Marketplaces in a Location-Specific Production Economy", *Regional Science and Urban Economics*, 30, 289–324.

[4] Bose, G., and M. Pingle (1995), "Stores", *Economic Theory,* 6, 251–262.

[5] Buck, R.C. (1978), *Advanced Calculus,* 3d edition, McGraw-Hill, New York.

[6] Debreu, G. (1959), *Theory of Value*, J. Wiley & Sons, New York.

[10] We thank a referee of this journal for pointing this out to us.

[7] Diamantaras, D., and R.P. Gilles (1996), "The Pure Theory of Public Goods: Efficiency, Decentralization and the Core", *International Economic Review,* 37, 851–860.

[8] Diamantaras, D., R.P. Gilles and S. Scotchmer (1996), "On the Decentralization of Pareto Optima in Economies with Public Projects, Nonessential Private Goods and Convex Costs," *Economic Theory,* 8, 555–564.

[9] Foley, D.K. (1970), "Economic Equilibrium with Costly Marketing", *Journal of Economic Theory,* 2, 276–291.

[10] Gilles, R.P. and D. Diamantaras (1998), "Linear cost sharing in economies with non-Samuelsonian public goods: Core equivalence", *Social Choice and Welfare,* 15, 121–139.

[11] Gilles, R.P., and D. Diamantaras (2003), "To Trade or Not to Trade: Economies with a Variable Number of Tradeables", *International Economic Review*, 44, 1173—1204.

[12] Gilles, R.P, Diamantaras, D., and P.H.M. Ruys (1994), "Efficiency and Separability in Economies with a Trade Center", *Working Paper E94-24,* Department of Economics, VPI&SU, Blacksburg.

[13] Halmos, P. (1950), *Measure Theory*, Reprint 1973, Springer Verlag, Berlin.

[14] Hammond, P. and A. Villar (1998), "Efficiency with non-convexities: Extending the "Scandinavian Consensus" approaches", *Scandinavian Journal of Economics* 100(1), 11–32.

[15] Hammond, P. and A. Villar (1999), "Valuation equilibrium revisited", in Alkan, A., C. Aliprantis, and O. Burkinshaw, *Current Trends in Economics*, Springer Verlag.

[16] Heller, W.P., and Starr, R. (1976), "Equilibrium with Non-convex use costs: Monetary and Non-monetary Economies", *Review of Economic Studies,* 43, 195–215.

[17] Hildenbrand, W. (1974), *Core and Equilibria of a Large Economy*, Princeton University Press, New Jersey.

[18] Kurz, M. (1974), "Equilibrium in a Finite Sequence of Markets with use cost", *Econometrica,* 42, 1–20.

[19] Laan, G. van der, P.H.M. Ruys, and D. Talman (2000), "Optimal provision of infrastructure in a public-private partnership relation", Discussion Paper # 2000-126, Center for Economic Research, Tilburg University, Tilburg.

[20] Loasby, B.J. (2000), "Market Institutions and Economic Evolution," *Journal of Evolutionary Economics*, 10, 297–309.

[21] Mas-Colell, A. (1980), "Efficiency and Decentralization in the Pure Theory of Public Goods", *Quarterly Journal of Economics,* 94, 625–641.

[22] Ménard, C. (1995), "Markets as Institutions versus Organizations as Markets? Disentangling some Fundamental Concepts," *Journal of Economic Behavior and Organization*, 28, 161–182.

[23] Ruys, P.H.M. (2002), "A general equilibrium of managed services", in: C. Hommes, R. Ramer and C. Withagen (eds.), *Equilibrium, Markets and Dynamics: Essays in the Honour of Claus Weddepohl*, Springer, Berlin, 117–139.

[24] Shubik, M. (1984), *A Game-Theoretic Approach to Political Economy,* MIT Press, Cambridge, Massachusetts.

[25] Yang, X. (2001), *Economics: New Classical versus Neoclassical Frameworks,* Blackwell Publishers, Malden, Mass.

[26] Yang, X. and Y.-K. Ng (1993), *Specialization and Economic Organization; A New Classical Microeconomic Framework*, North-Holland, Amsterdam.

[27] Zhou, L., G.-Z. Sun, and X. Yang (1999), "General Equilibria in Large Economies with use costs and Endogenous Specialization", typescript.

Appendices

A Proof of Lemma 3.4

Proof of (i).
The integrability result is immediate. The continuity on compact subsets of $\mathbb{R}^m_+$ follows on any compact subset of $\mathbb{R}^m_+$ immediately from a standard result on inverses of injective functions, for instance, Theorem 13 in Buck (1978, page 353) and is easily extended to continuity over all of $\mathbb{R}^m_+$, since continuity is a local property. Indeed, consider any point of $\mathbb{R}^m_+$ and take any compact set containing this point. Then by the continuity on compact sets, the function is continuous at this point, hence at all points; so the function is continuous on $\mathbb{R}^m_+$.

Proof of (ii).
This is a direct consequence of the definitions.

Proof of (iii).
Let $e = (1, \ldots, 1) \in \mathbb{R}^m_+$ be the vector of ones. Take $a \in A$ and $x \in \mathbb{R}^m_+$. Now define $y := g_\gamma(a, x+e) = x+e+t_\gamma(a, x+e)$. Then $n_\gamma(a, y) = n_\gamma(a, g_\gamma(a, x+e)) = x + e \gg x$.

Proof of (iv).
Let t_γ be monotone.

First, suppose that $x \geqq y$ and for some i: $n^i_\gamma(a, x) < n^i_\gamma(a, y)$. Then by monotonicity of t_γ we have that $t^i_\gamma(a, n_\gamma(a, x)) \leqq t^i_\gamma(a, n_\gamma(a, y))$. Hence,

$$\begin{aligned} x^i &= g^i_\gamma(a, n_\gamma(a, x)) = \\ &= n^i_\gamma(a, x) + t^i_\gamma(a, n_\gamma(a, x)) < \\ &< n^i_\gamma(a, y) + t^i_\gamma(a, n_\gamma(a, y)) = \\ &= g^i_\gamma(a, n_\gamma(a, y)) = y^i\ . \end{aligned}$$

This is a contradiction and thus we conclude that $n_\gamma(a, x) \geqq n_\gamma(a, y)$.

Second, let $x > y$. Then from the above we conclude that $n_\gamma(a, x) \geqq n_\gamma(a, y)$. Now suppose that $n_\gamma(a, x) = n_\gamma(a, y)$. Then $x = n_\gamma(a, x) + t_\gamma(a, n_\gamma(a, x)) = n_\gamma(a, y) + t_\gamma(a, n_\gamma(a, y)) = y$, which is a contradiction. This completes the proof of (iv), and thus of the lemma.

B Proof of Theorem 3.10

This proof is a modification of the proof of the first welfare theorem in Diamantaras and Gilles (1996).

Let (γ, f, y) be a valuation equilibrium in $\mathbb{E}_g$ with production plan ϕ with $\phi(\gamma) = y$, price system p and valuation function V. In order to show that $(\gamma, n_\gamma(f), y)$ is Pareto efficient in $\mathbb{E}$ it suffices to show that the gross allocation (γ, f, y) is Pareto efficient in $\mathbb{E}_g$.

Suppose to the contrary that (γ, f, y) is not Pareto efficient in $\mathbb{E}_g$. Then there exists a feasible allocation (δ, g, z) with for all a in A:

$$U_a(n_\delta(a, g(a))) \geqq U_a(n_\gamma(a, f(a))),$$

and there is a nonnegligible coalition E such that for all $b \in E$,

$$U_b(n_\delta(b, g(b))) > U_b(n_\gamma(b, f(b))).$$

Since (δ, g, z) is feasible it follows that

$$\int g\, d\mu + \int r_\delta\, d\mu + c(\delta) \leqq \int w\, d\mu + z. \tag{3}$$

Condition (v) of Definition 3.9 and the monotonicity of the utility functions imply that for all a in A we have $p(\delta) \cdot g(a) + p(\delta) \cdot r_\delta(a) + V(a, \delta) \geqq p(\delta) \cdot w(a)$ and for the agents $b \in E$ we have $p(\delta) \cdot g(b) + p(\delta) \cdot r_\delta(b) + V(b, \delta) > p(\delta) \cdot w(b)$. Hence,

$$p(\delta) \cdot \int g\, d\mu + p(\delta) \cdot \int r_\delta\, d\mu + \int V(\cdot, \delta)\, d\mu > p(\delta) \cdot \int w\, d\mu. \tag{4}$$

Since equation (3) can be written as

$$z + \int (w - r_\delta - g)\, d\mu - c(\delta) \geqq 0, \tag{5}$$

we have:

$$p(\delta) \cdot z + p(\delta) \cdot \int (w - r_\delta - g)\, d\mu - p(\delta) \cdot c(\delta) \geqq 0. \tag{6}$$

Combining (4) with (6), we have:

$$p(\delta) \cdot z - p(\delta) \cdot c(\delta) \geqq -p(\delta) \cdot \int (w - r_\delta - g)\, d\mu > -\int V(\cdot, \delta)\, d\mu,$$

which leads to:

$$p(\delta) \cdot z + \int V(\cdot, \delta)\, d\mu - p(\delta) \cdot c(\delta) > 0. \tag{7}$$

Conditions (iii) and (iv) of Definition 3.9 now implies that

$$0 = \int V(\cdot, \gamma)\, d\mu - p(\gamma) \cdot c(\gamma) \geqq \int V(\cdot, \delta)\, d\mu - p(\delta) \cdot c(\delta). \tag{8}$$

Now, from condition (i),

$$0 = p\,(\gamma) \cdot y = p(\delta) \cdot \phi(\delta) \geqq p(\delta) \cdot z. \tag{9}$$

Combining (7), (8), and (9), we obtain:

$$0 = p\,(\gamma) \cdot y + \int V\,(\cdot, \gamma)\, d\mu - p(\gamma) \cdot c(\gamma) \geqq p(\delta) \cdot z + \int V\,(\cdot, \delta)\, d\mu - p(\delta) \cdot c(\delta) > 0,$$

with the last inequality because of (7).
This is a contradiction proving the assertion.

C Proof of Theorem 3.12

This proof is based on the structure of the proof of the second welfare theorem in Diamantaras, Gilles and Scotchmer (1996). However, some major modifications are called for. The main difference from that paper is that we have non-convexities in preferences. As a result, our theorem is stated for a continuum of agents, requiring substantial preparatory work and the use of Hildenbrand's (1974) results regarding convexification in a large economy.

Before we show Theorem 3.12 we prove two lemmata regarding some mathematical properties of integrals of correspondences. The first concerns the openness of the integral of an open valued correspondence.

Lemma A.1. *Let $n \in \mathbb{N}$ and let F be an open valued and measurable correspondence from A to $\mathbb{R}^n$. Then $\int F\, d\mu$ is an open subset of $\mathbb{R}^n$.*

Proof. If $\int F\, d\mu = \emptyset$, the assertion is true by definition. Hence, suppose that $\int F\, d\mu \neq \emptyset$.

Now, for every $\varepsilon > 0$ define

$$B_\varepsilon := \{x \in \mathbb{R}^n \mid \|x\| < \varepsilon\}$$

as the open ε-ball at the origin in $\mathbb{R}^n$.

Let $\hat{x} \in \int F\, d\mu$. Then by definition there exists an integrable function $f : A \to \mathbb{R}^n$ with $f(a) \in F(a)$, $a \in A$, and $\int f\, d\mu = \hat{x}$.

Since F is open valued we can define the function $\delta : A \to \mathbb{R}_{++}$ given by

$$\delta(a) := \inf \{\|f(a) - x\| \mid x \in \mathbb{R}^n \setminus F(a)\}, \quad a \in A.$$

Since F is measurable and f integrable, the function δ is measurable. By definition $\delta(a) > 0$ a.e. on A and $\{f(a)\} + B_{\delta(a)} \subset F(a)$ a.e. on A.

Now let $0 < k < 1$. Recall that the probability space (A, Σ, μ) is a regular Borel measure space based on the locally compact Hausdorff space $(A, \mathcal{T})$. Thus, by Lusin's theorem (Halmos, 1950, page 243) there exists a $\mathcal{T}$-compact set $C \subset A$ with $\mu(A \setminus C) < k$ and δ continuous on C.

Since C is compact, δ is in fact uniformly continuous on C, and we may define $\varepsilon := \frac{1}{2} \min_{a \in C} \delta(a) > 0$. Thus by definition for every $a \in C$:

$$\{f(a)\} + B_\varepsilon \subset F(a) \tag{10}$$

Define $\gamma := \mu(C)$. Clearly $0 < 1 - k < \gamma < 1$. We will now show that

$$\hat{x} + B_{\gamma\varepsilon} \subset \int F\, d\mu \tag{11}$$

Take $y \in B_{\gamma\varepsilon}$. Then $\frac{1}{\gamma}\, y \in B_\varepsilon$. Now we define the function $g : A \to \mathbb{R}^n$ by

$$g(a) := \begin{cases} f(a) + \frac{1}{\gamma}\, y & a \in C \\ f(a) & a \in A \setminus C \end{cases}$$

Then g is an integrable selection of F with

$$\int g\,d\mu = \int f\,d\mu + \frac{1}{\gamma}\,y\,\mu(C) = \hat{x} + y \in \int F\,d\mu.$$

Hence, we have established (11). ■

The second lemma addresses the support of convex integrals of correspondences by hyperplanes. This refers to the existence of optimal consumption bundles in budget sets with a certain supporting price.

Lemma A.2. *Let $n \in \mathbb{N}$ and let F be a measurable correspondence from (A, Σ, μ) into $\mathbb{R}^n$ such that for every $a \in A$ the set $F(a)$ is closed, for every $x \in F(a)$: $\{x\} + \mathbb{R}^n_+ \subset F(a)$, and there exists an integrable function $b : A \to \mathbb{R}^n$ with $b(a)$ a lower bound for $F(a)$, a.e. on A. If $\int F\,d\mu \neq \emptyset$, then for every vector $p \in \mathbb{R}^n_{++}$ there exists an integrable selection g of F such that*

(a) *$\int g\,d\mu$ is on the boundary of $\int F\,d\mu$ such that $p \cdot \int g\,d\mu = \inf\, p \cdot \int F\,d\mu$ and*

(b) *for almost every $a \in A$ it holds that $p \cdot g(a) = \inf\, p \cdot F(a)$.*

Proof. First note that since F is bounded from below by b and measurable, $\int F\,d\mu$ is bounded from below by $\int b\,d\mu$. Furthermore, since $\int F\,d\mu \neq \emptyset$, there exists an integrable selection $f : A \to \mathbb{R}^n$ of F.
Let $p \in \mathbb{R}^n_{++}$ be a strictly positive vector. Then by Proposition D.6 of Hildenbrand (1974, page 63) the function $h : A \to \mathbb{R}$ given by $h(a) := \inf\{p \cdot x \mid x \in F(a)\}$ is integrable and

$$\inf\, p \cdot \int F\,d\mu = \int h\,d\mu \geqq p \cdot \int b\,d\mu. \tag{12}$$

Now consider the correspondence H from A into $\mathbb{R}^n$ given by

$$H(a) := \{x \in F(a) \mid p \cdot x \leqq p \cdot f(a) + 1\}, \tag{13}$$

where $a \in A$. By measurability of F it immediately follows that H is measurable. Moreover, by comprehensiveness, boundedness and closedness of $F(a)$, $a \in A$, H takes almost everywhere nonempty and compact values. From its construction it is easy to see that H is integrably bounded.
Now the mapping X on A into $\mathbb{R}^n$ defined by

$$X(a) := \{x \in F(a) \mid p \cdot x = h(a)\} \subset H(a), \quad a \in A, \tag{14}$$

is measurable by the integrability of h. Since $F(a)$ is closed and bounded from below and $p \gg 0$, it is concluded that $X(a) \neq \emptyset$ for almost every $a \in A$. Since $X(a) \subset H(a)$ for any $a \in A$ and H is integrably bounded, it follows that X is integrably bounded also. Thus, by, e.g., Corollary 17.1.4, Klein and Thompson (1984, page 186), X is Aumann-integrable and by Aumann's selection theorem there exists an integrable selection g in X. By display (12) we conclude that

$$p \cdot \int g \, d\mu = \int h \, d\mu = \inf p \cdot \int F \, d\mu.$$

This shows that $\int h \, d\mu \in \partial \int F \, d\mu$ and thus we have shown (a).

By definition of X it follows that $p \cdot g(a) = h(a) = \inf p \cdot F(a)$ for almost every $a \in A$. Thus, we have shown (b). ■

Proof of Theorem 3.12 (a)

Let $\left(\gamma, \breve{f}, y\right)$ be a Pareto efficient allocation in $\mathbb{E}$, so its gross representation given by (γ, f, y) with $f = g_\gamma\left(\breve{f}\right)$ is Pareto efficient in $\mathbb{E}_g$. We now show that (γ, f, y) can be supported as a valuation quasi-equilibrium in $\mathbb{E}_g$. Let $a \in A$ and $\delta \in \Gamma$ be arbitrary. We define

$$\begin{aligned} F(a,\delta) &= \left\{ g \in \mathbb{R}^m_+ \,|U_a(n_\delta(a,g)) > U_a(n_\gamma(a, f(a))) \right\}, \quad \text{and} & (15)\\ \overline{F}(a,\delta) &= \left\{ g \in \mathbb{R}^m_+ \,|U_a(n_\delta(a,g)) \geqq U_a(n_\gamma(a, f(a))) \right\}. & (16) \end{aligned}$$

Note that $F(a,\delta) \neq \emptyset$ by monotonicity of U_a and the monotonicity properties in Lemma 3.4 regarding n_γ and n_δ. Moreover, it follows from continuity of U_a and the net consumption functions that $F(a,\delta)$ is open and bounded from below by 0, but not necessarily convex. By monotonicity of U_a it holds that for any $x \in F(a,\delta)$: $\{x\} + \mathbb{R}^m_+ \subset F(a,\delta)$. Finally, $\overline{F}(a,\delta)$ is the closure of $F(a,\delta)$, by the continuity of U_a, and, except for being a closed set, $\overline{F}(a,\delta)$ inherits all the properties of $F(a,\delta)$ listed.

Furthermore, from the measurability condition on the collection of utility functions $\{U_a\}_{a\in A}$ we conclude that $F(\cdot,\delta)$ as well as $\overline{F}(\cdot,\delta)$ are measurable correspondences. Let

$$F(\delta) := \int F(\cdot,\delta) \, d\mu + \left\{ \int r_\delta \, d\mu + c(\delta) - \int w \, d\mu \right\}. \tag{17}$$

First we show that $F(\delta) \neq \emptyset$. The nonemptiness of $F(\delta)$ follows from the monotonicity properties of n_δ, the monotonicity of the preferences, and the uniform boundedness of the use cost functions. This is shown as follows: first, note that for every $a \in A$ by monotonicity of use costs and Lemma 3.4, $n_\gamma(a, f(a) + e) > n_\gamma(a, f(a))$, where $e = (1, \ldots, 1) \in \mathbb{R}^m_+$, and thus by strict monotonicity of the utility functions $g(a) \in \overline{F}(a,\delta)$, where $g(a) \in \mathbb{R}^m_+$ is defined by

$$g(a) := n_\gamma\left(a, f(a) + e\right) + t_\delta\left(a, n_\gamma\left(a, f(a) + e\right)\right) \tag{18}$$

Hence,

$$n_\delta(a, g(a)) = n_\gamma(a, f(a) + e) > n_\gamma(a, f(a)).$$

Note that $g : A \to \mathbb{R}^m_+$ is measurable by the integrability and continuity properties of n_γ and t_δ. Next we show that g is integrably bounded, and thus integrable. Namely by the boundedness of use costs there exists a $K > 0$ such that $t_\delta(a, x) \leqq K\, x$ for every $a \in A$ and, thus, by definition of $g(a)$ we have

$$\begin{aligned} g(a) &= n_\gamma(a, f(a) + e) + t_\delta(a, n_\gamma(a, f(a) + e)) \\ &\leqq f(a) + e + t_\delta(a, f(a) + e) \\ &\leqq f(a) + e + K(f(a) + e) = (K + 1)(f(a) + e) \end{aligned}$$

Since f is integrable, this implies that g is integrably bounded and thus integrable. Thus $\int g\, d\mu \in \int F(\cdot, \delta)\, d\mu$, which implies that $F(\delta) \neq \emptyset$.

Second, because (A, Σ, μ) is atomless, it follows by Liapunov's theorem that $F(\delta)$ is convex. Furthermore, $F(\delta)$ is bounded from below. Also, from Lemma A.1 it follows that $F(\delta)$ is open. Finally, we conclude that because (γ, f, y) is efficient, we have $Y \cap F(\delta) = \emptyset$.

By Minkowski's separating hyperplane theorem, e.g., Hildenbrand (1974, page 38), applied to Y and $F(\delta)$, there exists a vector $p(\delta) \in \Delta$ such that

$$p(\delta) \cdot v \geqq p(\delta) \cdot y' \text{ for all } v \in F(\delta) \text{ and all } y' \in Y.$$

Since $F(\delta)$ is an open set and Y a pointed cone containing the origin, this implies that $p(\delta) \cdot v > 0$ for all $v \in F(\delta)$. (The nonnegativity of $p(\delta)$ follows from the assumption of free disposal on Y.)

In this fashion we have defined a function $p : \Gamma \to \Delta$. We now show that p satisfies the conditions required by Definition 3.11.

Condition (i).
Construct a production plan $\phi : \Gamma \to Y$ by selecting for every $\delta \neq \gamma$ one $\phi\,(\delta) \in Y$ such that $p(\delta) \cdot \phi\,(\delta) = 0$. (Because Y is a convex cone containing the origin and satisfying free disposal and $p(\delta)$ is a nonnegative vector, such an $\phi\,(\delta)$ indeed exists.) Furthermore, we let $\phi\,(\gamma) = y$. Note that from the definition of the set $F\,(\gamma)$ and the feasibility of the allocation (γ, f, y) it holds that $y \in \overline{F}\,(\gamma)$, where $\overline{F}\,(\gamma)$ is the closure of the set $F\,(\gamma)$. From this we immediately conclude that $0 \leqq p\,(\gamma) \cdot y \leqq 0$, i.e., $p\,(\gamma) \cdot \phi\,(\gamma) = p\,(\gamma) \cdot y = 0$. Condition (i) now follows from the separation argument above.

Let $x(a, \delta) \in \mathbb{R}_+$ be defined such that

$$x(a, \delta) := \begin{cases} \inf\ p(\delta) \cdot F(a, \delta) & \text{if } \delta \neq \gamma \\ \quad p(\gamma) \cdot f(a) & \text{if } \delta = \gamma \end{cases}$$

By integrability of infima as stated in, e.g., Proposition 18.1.8 in Klein and Thompson (1984, page 200), it follows that for every $\delta \in \Gamma$ the function $x(\cdot, \delta) : A \to \mathbb{R}$ is integrable and

$$\int x(\cdot, \delta)\, d\mu = \inf p(\delta) \cdot \int F(\cdot, \delta)\, d\mu.$$

Hence,

$$\int x(\cdot,\delta)\,d\mu + p(\delta)\cdot\int r_\delta\,d\mu + p(\delta)\cdot c(\delta) - p(\delta)\cdot\int w\,d\mu = \inf p(\delta)\cdot F(\delta) \geqq 0.$$

Finally, we introduce the valuation function $V : A\times\Gamma \to \mathbb{R}$ by

$$\begin{aligned} V(a,\delta) &= p(\delta)\cdot w(a) - p(\delta)\cdot r_\delta(a) - x(a,\delta), \quad \delta\neq\gamma, \quad \text{and} \\ V(a,\gamma) &= p(\gamma)\cdot w(a) - p(\gamma)\cdot r_\gamma(a) - p\,(\gamma)\cdot f\,(a)\,. \end{aligned}$$

Note that $V(a,\delta)$ is finite for almost every agent a in A and every $\delta\in\Gamma$.

We now check the remaining requirements of Definition 3.11.

Condition (ii)

Let $\delta\in\Gamma$. Since $x(\cdot,\delta)$ and f are integrable on A, the valuation function as introduced above is also integrable. Furthermore, since $F(a,\delta)\subset\mathbb{R}^m_+$ and $p(\delta)\geqq 0$, we conclude that $x(a,\delta)\geqq 0$ for every $a\in A$. Thence, by definition $V(a,\delta)\leqq p(\delta)\cdot w(a) - p(\delta)\cdot r_\delta(a)$ for almost every $a\in A$ and all $\delta\in\Gamma$.

Condition (iii)

By the feasibility of (γ,f,y) and the definition of V,

$$\begin{aligned} \int V(\cdot,\gamma)\,d\mu &= p(\gamma)\cdot\int w\,d\mu - p(\gamma)\cdot\int r_\gamma\,d\mu - p(\gamma)\cdot\int f\,d\mu \\ &\geqq p(\gamma)\cdot c(\gamma) - p\,(\gamma)\cdot y = p(\gamma)\cdot c(\gamma). \end{aligned}$$

If the second relation is not an equality, then there exists a non-negligible set of agents to whom we can feasibly redistribute some private goods to make them better off, because of the strict monotonicity of the utility functions and the monotonicity of use costs (Lemma 3.4). This contradicts efficiency, hence we have $p(\gamma)\cdot\int w\,d\mu - p(\gamma)\cdot\int r_\gamma\,d\mu - p(\gamma)\cdot\int f\,d\mu = p(\gamma)\cdot c(\gamma)$.

Condition (iv)

By construction, in case $\delta\neq\gamma$:

$$\int x(\cdot,\delta)\,d\mu + p(\delta)\cdot\int r_\delta\,d\mu + p(\delta)\cdot c(\delta) \geqq p(\delta)\cdot\int w\,d\mu.$$

From this we obtain

$$\int V(\cdot,\delta) = p(\delta)\cdot\int w\,d\mu - p(\delta)\cdot\int r_\delta\,d\mu - \int x(\cdot,\delta)\,d\mu \leqq p(\delta)\cdot c(\delta).$$

Thus, together with (iii) as shown above we conclude that condition (iv) of Definition 3.11 is satisfied for the price vector $p(\delta)$ and valuation $V(\cdot,\delta)$, $\delta\in\Gamma$.

Condition (v')

First note that by definition of $V(a,\gamma)$ it immediately follows that

$$p(\gamma)\cdot f(a)+p(\gamma)\cdot r_\gamma(a)+V(a,\gamma)=p(\gamma)\cdot w(a).$$

Now let $\delta\neq\gamma$ and $g\in\mathbb{R}^m_+$ such that $\bar{U}_a(g,\delta)>\bar{U}_a(f(a),\gamma)$. Then by definition $g\in\overline{F}(a,\delta)$ and, since $\overline{F}(a,\delta)$ is the closure of $F(a,\delta)$, $p(\delta)\cdot g\geqq x(a,\delta)$. Thus,

$$p(\delta)\cdot g+p(\delta)\cdot r_\delta(a)+V(a,\delta)\geqq p(\delta)\cdot w(a).$$

This shows condition (v') for valuation quasi-equilibrium.

Proof of Theorem 3.12 (b)
By Theorem 3.12(a) shown above the Pareto efficient allocation (γ,f,y) can be supported as a valuation quasi-equilibrium defined in Definition 3.11. To show assertion (b) we only have to show that, under the given conditions, property (v) of Definition 3.9 is satisfied.

Let $\phi:\Gamma\to Y$ be the production plan, $p:\Gamma\to\Delta$ be the price system and $V:A\times\Gamma\to\mathbb{R}$ the valuation system as constructed in the proof of Theorem 3.12(a) given above. Now define for every $a\in A$

$$B(a,p,V):=\left\{(g,\delta)\in\mathbb{R}^m_+\times\Gamma\;\middle|\;p(\delta)\cdot g+p(\delta)\cdot r_\delta(a)+V(a,\delta)\leqq p(\delta)\cdot w(a)\right\}$$

as a's budget set. It is our goal to show that $(f(a),\gamma)$ is $\bar{U}_a$ maximal in $B(a,p,V)$.

Let $\delta\in\Gamma$. First we show that $p(\delta)\gg 0$. Define for every $a\in A$

$$B_\delta(a):=\{g\in\mathbb{R}^m_+\mid p(\delta)\cdot g+p(\delta)\cdot r_\delta(a)+V(a,\delta)\leqq p(\delta)\cdot w(a)\}\quad\text{and}$$

$$B'_\delta(a):=\{g\in\mathbb{R}^m_+\mid p(\delta)\cdot g+p(\delta)\cdot r_\delta(a)+V(a,\delta)<p(\delta)\cdot w(a)\}.$$

By Axiom 2.4 $\int r_\delta\,d\mu+c(\delta)\ll\int w\,d\mu+z$ for some $z\in Y$. Since $p(\delta)>0$ and $p\,(\delta)\cdot z\leqq 0$ it follows that

$$p(\delta)\cdot\int r_\delta\,d\mu+\int V(\cdot,\delta)\,d\mu<p(\delta)\cdot\int w\,d\mu+p\,(\delta)\cdot z\leqq p(\delta)\cdot\int w\,d\mu.$$

This implies that there exists a nonnegligible set $E\in\Sigma$, $\mu(E)>0$, such that for every $a\in E:\ p(\delta)\cdot[w(a)-r_\delta(a)]>V(a,\delta)$. Thus, we may conclude that $B'_\delta(a)\neq\emptyset$ for every agent $a\in E$.

Let $a\in E$. From the proof of property 3.11(v') given in the proof of Theorem 3.12(a), it is immediately deduced that for all $g\in B'_\delta(a)$ it holds that $\bar{U}_a(g,\delta)\leqq\bar{U}_a(f(a),\gamma)$. Now take $g\in B_\delta(a)$, then there exists a sequence $(g_n)_{n\in\mathbb{N}}$ in $B'_\delta(a)$ with $g_n\to g$. By continuity of $\bar{U}_a(\cdot,\delta)$ this implies that $\bar{U}_a(g,\delta)\leqq\bar{U}_a(f(a),\gamma)$. Thus, for every $g\in B_\delta(A):\ \bar{U}_a(g,\delta)\leqq\bar{U}_a(f(a),\gamma)$.

Now suppose $p^i(\delta)=0$ for some $i\in\{0,1,\ldots,\ell\}$. We claim that for every $x>0$ there exists a bundle $h\in B_\delta(a)$ with $n^i_\delta(a,h)>x$.

Indeed let $\hat{h} \in B_\delta(a)$ be such that $\hat{h}^i > 0$. Now define $h := \hat{h} + (K+1)x\, e^i \in B_\delta(a)$, where e^i is the i-th unit vector and K is the given bound for the use costs.

Now for any $h' \in \mathbb{R}^m_+$ it holds that

$$g_\delta(a,h) = h' + t_\delta(a,h') \leqq (K+1)h' \tag{19}$$

by the boundedness of the use costs. Hence, by the monotonicity properties of n_δ, $h' = n_\delta\left(a, g_\gamma(a,h')\right) \leqq n_\delta\left(a, (K+1)h'\right)$.
With the definition of h above this leads to

$$n^i_\delta\left(a,h\right) = n^i_\delta\left(a, \hat{h} + (K+1)x\, e^i\right) \geqq \frac{1}{K+1}\hat{h}^i + x > x.$$

Since U_a has compact lower contour sets, $\{x \in \mathbb{R}^m_+ \mid U_a(x) \leqq \bar{U}_a(f(a),\gamma)\}$ is compact, and therefore bounded. But $p^i(\delta) = 0$ together with the claim implies that there exists a bundle $g \in B_\delta(a)$ with $\bar{U}_a(g,\delta) = U_a(n_\delta(a,g)) > \bar{U}_a(f(a),\gamma)$. This contradicts that for $g \in B_\delta(A)$ $\bar{U}_a(g,\delta) \leqq \bar{U}_a(f(a),\gamma)$ as derived above. Since this can be repeated for any $a \in E$ and E is nonnegligible, this implies that $p(\delta) \gg 0$.

The argument above can be repeated for every $\delta \in \Gamma$, deriving that p is a strictly positive price system. Now let $\delta \neq \gamma$. Recall that for every $a \in A$ we defined

$$\overline{F}(a,\delta) := \left\{g \in \mathbb{R}^m_+ \mid U_a(n_\delta(a,g)) \geqq U_a(n_\gamma(a,f(a)))\right\}.$$

Noting that the correspondence $\overline{F}\left(\cdot,\delta\right)$ and the price vector $p\left(\delta\right) \gg 0$ satisfies all requirements, we may apply Lemma A.2, implying the existence of an integrable selection $g\left(\cdot,\delta\right) : A \to \mathbb{R}^m_+$ such that for almost every $a \in A$: $p\left(\delta\right) \cdot g\left(a,\delta\right) = \inf p\left(\delta\right) \cdot \overline{F}(a,\delta) = x\left(a,\delta\right)$ and

$$p\left(\delta\right) \cdot \int g\left(\cdot,\delta\right)\, d\mu = \inf\, p\left(\delta\right) \cdot \int \overline{F}(\cdot,\delta)\, d\mu$$

For γ we introduce $g\left(\cdot,\gamma\right) : A \to \mathbb{R}^m_+$ by $g\left(a,\gamma\right) = f\left(a\right)$, $a \in A$. Thus, for any $\delta \in \Gamma$ we can reformulate $V\left(\cdot,\delta\right)$ by

$$V\left(a,\delta\right) = p\left(\delta\right) \cdot \left[w\left(a\right) - r_\delta\left(a\right) - g\left(a,\delta\right)\right].$$

Furthermore, from continuity of preferences U_a of $a \in A$: $U_a(n_\delta(a, g\left(a,\delta\right))) = U_a(n_\gamma(a,f(a)))$. We now check condition (v) of Definition 3.9.

From condition (v) of Definition 3.11 shown above in the proof of Theorem 3.12(a) for ϕ, p and V it immediately follows that $f\left(a\right)$ is $\bar{U}_a$-maximal in $B_\gamma\left(a\right)$. Next let $\delta \neq \gamma$. Then by definition $g\left(a,\delta\right) \in B_\delta\left(a\right)$. Now for $x \in \mathbb{R}^m_+$ with $U_a\left(n_\delta\left(a,x\right)\right) > U_a(n_\gamma(a,f(a))) = U_a(n_\delta(a, g\left(a,\delta\right)))$ we have by definition of $g\left(a,\delta\right)$ as the supporting point at $F\left(a,\delta\right)$ of the hyperplane defined by $p\left(\delta\right)$ that $p\left(\delta\right) \cdot x > p\left(\delta\right) \cdot g\left(a,\delta\right)$. Thus, by definition of $V\left(a,\delta\right)$ this implies that $x \notin B_\delta\left(a\right)$. Thus, we conclude that $g\left(a,\delta\right)$ is $\bar{U}_a$-maximal in $B_\delta\left(a\right)$. This implies that $(f(a),\gamma)$ is $\bar{U}_a$-maximal in $B(a,p,V)$, showing condition (v) of Definition 3.9.

This completes the proof of Theorem 3.12.